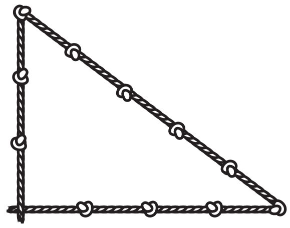
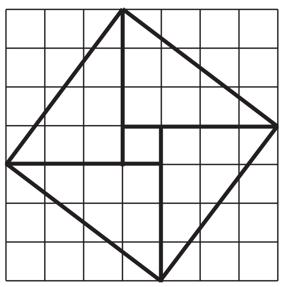
The word **geometry** comes from the Greek Geo-earth, Metria- measurement. The Egyptians used geometry to **draw** over their land the **shapes** of their fertile fields by the Nile river. Once a year, the Nile increased its level of water which obscured the outlines of the plots.



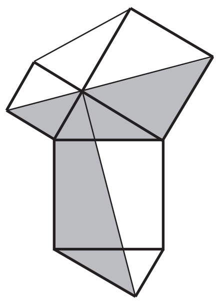
The Egyptians used a rope, divided into twelve parts with eleven knots, to draw over and over their fields. Forming with the string a **triangle**, whose **sides** measure three, four and five parts of the rope, they got a **right triangle** that helped them to draw the plots on the land in an orderly manner with **right angles** and **perpendicular and parallel lines**. **Pythagoras** learned this trick from the Egyptians five hundred years before Christ, and today this geometry is still used for science and progress.

Throughout history we find different **graphic** demonstrations of Pythagoras **theorem** based on the right triangle. The Chineese graphic **demonstration**, very elegant and pretty, belongs to the same era than Pythagoras and it is believed that he did not know it. Approximately two hundred years after Pythagoras, Euclid, Ancient Greek **Philosopher**, also **drew** a demonstration. Two thousand years later, Leonardo da Vinci drew another demonstration.

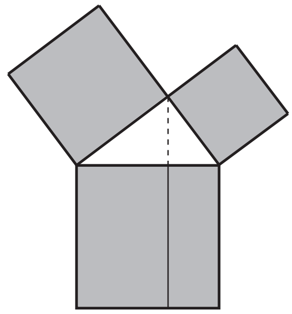
Chinese demonstration



Leonardo demonstration



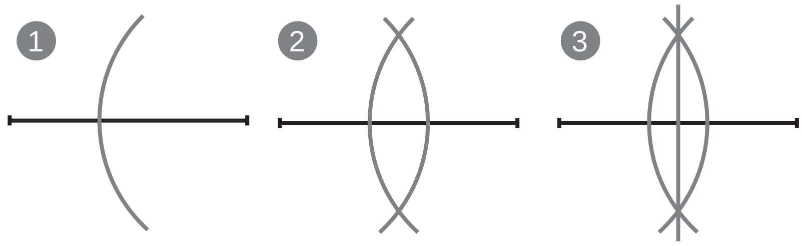
Euclides demonstration

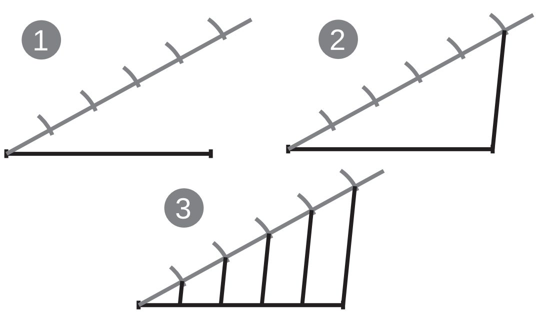


DIVIDING A SEGMENT INTO EQUAL PORTIONS

Perpendicular segment bisector:

1. With any radius bigger than half of the given segment trace an arc with center in one of the end points.
2. With the same radius trace an arc with center in the other end point.
3. Join the two intersection points of the two arcs with a straight line. This must cut perpendicularly the segment in two equal parts.

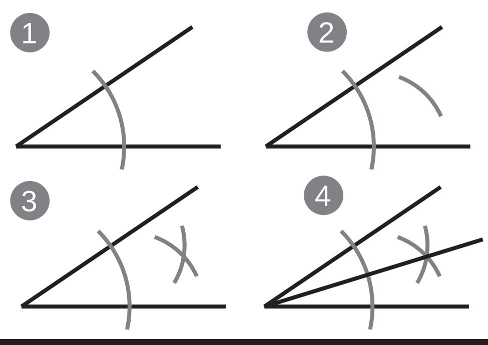




Divide the segment in (5)equal portions using the Thales theorem.

1. Forming any angle with the given segment, trace from one of its end points a straight line. Divide the line in five equal parts which have the same length
2.Conect the last division with the opposite end point of the given segment.
3. Through every division left trace parallels to the line traced in the previous step. All these parallels divide the given segment in five equal parts.

DIVIDING AN ANGLE INTO TWO EQUAL PORTIONS

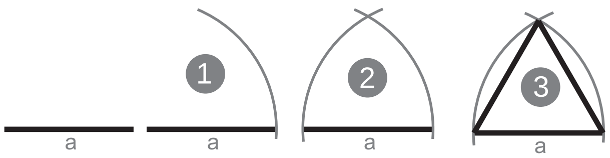


Divide the angle in 2 equal portions. Angle bisector:

1. With center in the vertex of the given angle trace an arc that cuts both sides of the angle in two points.
2. Choose a radius and with center in one of the points produced by the arc trace another arc.
3. With the same radius trace another arc with center in the other point produced by the first arc. Both arcs cut themselves in a point.
4. Conect the point produced by the last two arcs with the vertex of the angle with a straight line.

Regular polygons construction given the side a:

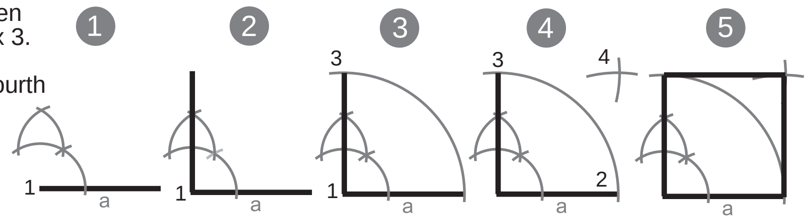
Equilateral Triangle



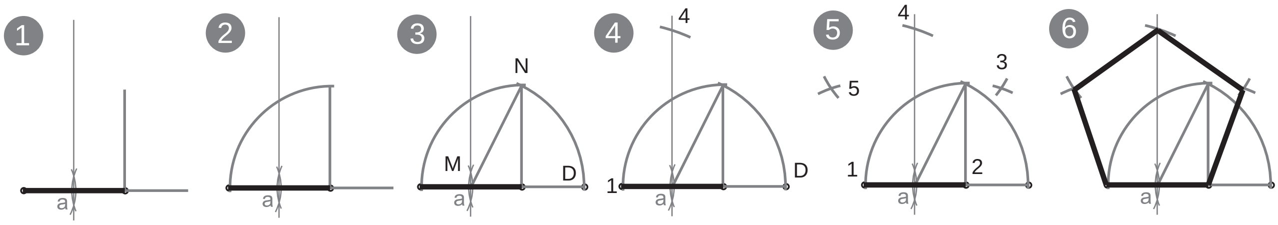
1st- With center in an end point trace an arc with the same radius as the segment's length.
2nd- With center in the other endpoint repeat the same step.
3rd- The point where both arcs meet is the third vertex of the equilateral triangle. conect it with both segments' end points.

Square

1st- With the compass setting the first center in vertex 1, trace four arcs with the same radius that define four points.
2nd- Join the fourth point with vertex 1.
3rd- With center in vertex 1 and radius equal to the given side trace an arc that cuts the vertical line in vertex 3.
4th- With radius equal to the given side and center in points 3 and 2, trace two arcs that intersect in the fourth vertex.
5th- Conect vertices 3 and 2 with 4.

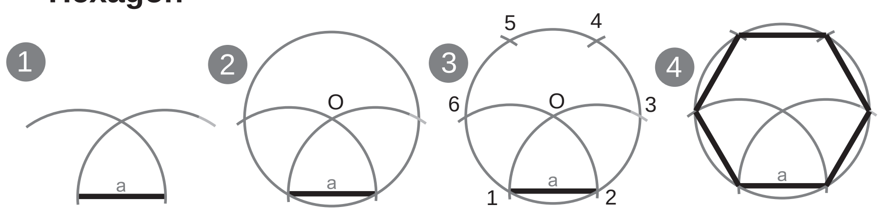


Pentagon



1st- Trace the perpendicular bisector of a. Through the end point on the right trace a perpendicular line and extend the given side.
2nd- With center on the left end point and radius equal to the given side trace an arc that meets the perpendicular line traced.
3rd- With center in the given segment's middle point and radius MN trace an arc that cuts the segment's extension in D point.
4th- With center in vertex 1 and radius 1D trace an arc that meets the perpendicular bisector traced in point 4.
5th- With radius equal to the given side trace arcs with center in 1, 2 and 4 to obtain vertices 3 and 5.
6th- conect the five vertices to obtain the Pentagon.

Hexagon

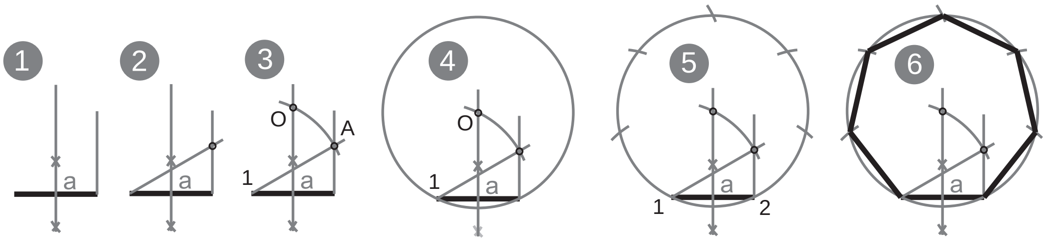


1st- With radius equal to the given side trace to arcs that intersect in O.
2nd- With center in O and a radius equal to the given side trace a circle through both given end points.
3rd- With center in 3 and and a radius equal to the given side trace two arcs that meet the circle in points 4 and 5.
4th- Join the six vertices.

Heptagon

1st- Trace the perpendicular segment bisector and raise a perpendicular line through an end point.

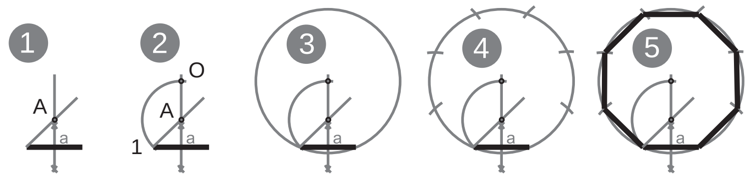
2nd- Trace a 30º angle through the other end point.
3rd- With center in point 1 and radius 1A trace an arc that meets the perpendicular bisector in point O.



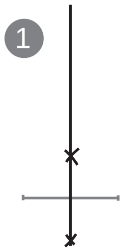
4th-With ceter in O and a radius O1 trace a circumference which will enclose (circumscribe) the heptagon.
5th- With a radius equal to the given side, starting by 1 and 2, trace arcs which intersect the circumference in the vertices 3,4,5,6 and 7
6th- Join the seven vertices.

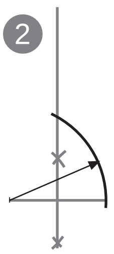
Octagon

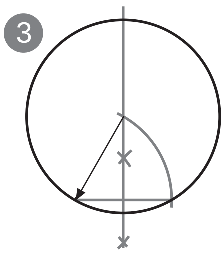
1st- Trace the perpendicular segment bisector and from an endpoint trace 45º angle to obtain A over the perpendicular bisector.
2nd- With center in A and radius A1 trace an arc that intersects the perpendicular bisector in O.
3rd- With center in O and a O1 radius trace a circumference.
4th- With radius equal to the given side trace arcs over the circumference which will show us the six vertices left.
5th- Join the eight vertices together.



n(9) sides regular polygon construction given one side:

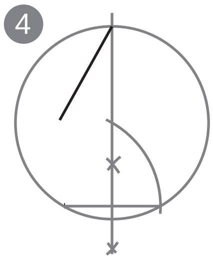


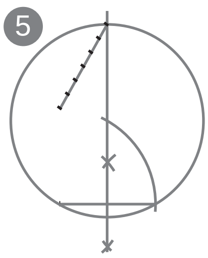


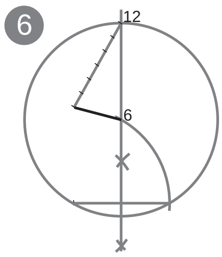


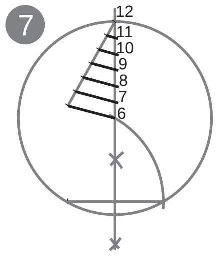
1st-Trace the given side's perpendicular bisector.
2nd- Centered in one side's end point and a radius equal to it trace an arc that intersects the perpendicular bisector.
3rd- Centered in the intersection of the arc and the perpendicular bisector and a radius equal to the given side trace a circle which will go thorugh both side's endpoints.

Make sure the perpendicular bisector intersects the circle by its upper part. That way the perpendicular bisector is a circumference diameter. After this consideration divide, with Thales method, the upper radius of the diameter into six equal portions:



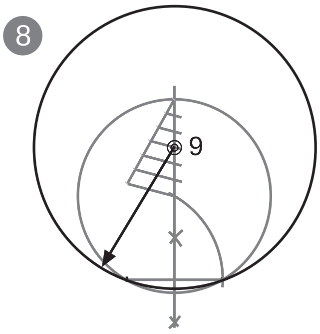




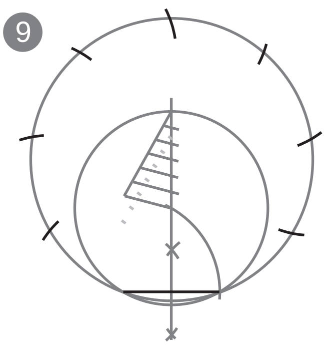


4th- Trace an auxiliary segment making an angle with the diameter with the vertex in its upper endpoint.
5th- Divide the auxiliary segment in six equal portions (with a compas).
6th- Conect the last auxiliary segment's division with the circle's center. Label the center as number 6 and the top diameter's endpoint as number 12.
7th- Trace paralels through the divisions obtaining that way the divisions wanted on the upper diameter's radius.

In this case we are aiming for an Nonagon (9 sides polygon). So we will set the center for a circunscribed circle in the division number 9. If we were looking for a regular polygon with a different number of sides we would set the center in the division named with the number of sides wanted.

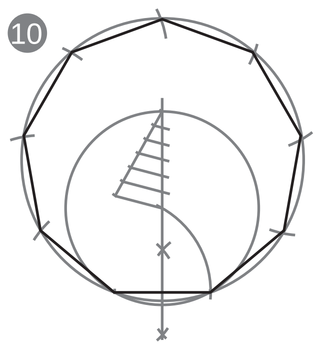


8th- With center in the division number 9 and opening the compass up to any of the given side's endpoints, trace a circle. This circumference must go through the other segment's endpoint.



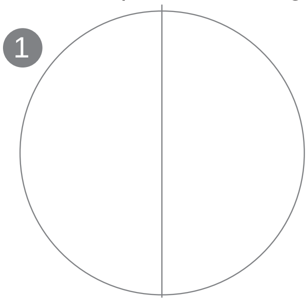
9th- Take the given side's length with the compas and repeat arcs with that radius on the bigger circle starting by the given side's endpoint.

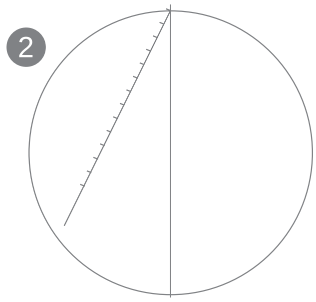
10th- Finally conect the points marked by the arcs intersecting the circle to obtain the wanted regular polygon.

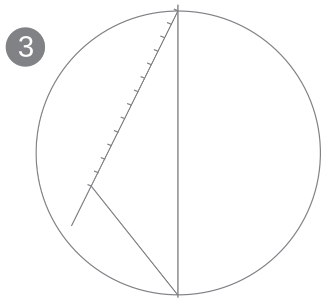


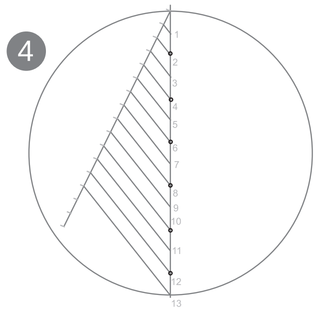
Given the circumscribed circle radius a: Draw a n sided (13) regular polygon: ________

1th- Trace a circle with the given radius lengh, Trace a vertical diameter:
DIVIDE THE DIAMETER INTO AS MANY PORTIONS AS AIMED SIDES FOR THE POLYGON WANTED
2nd- Trace an auxiliary segment forming any angle with the vertex in the top diameter's end point and divide it into as many portions as wanted portions for the diameter (you can use either a ruler or the compass)
3rd- Conect the last auxiliary segment's mark with the bottom diameter's end point.
4th- Trace parallels through the division marks meeting the diameter obtaining the divisions wanted on it.

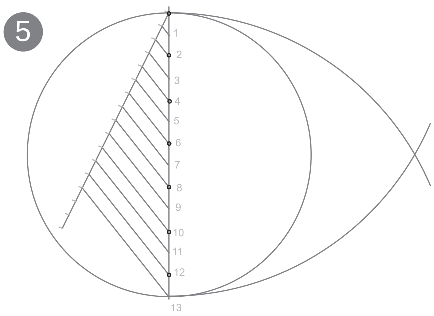


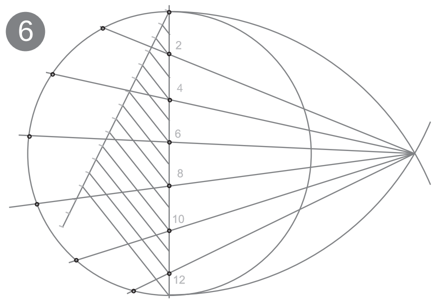




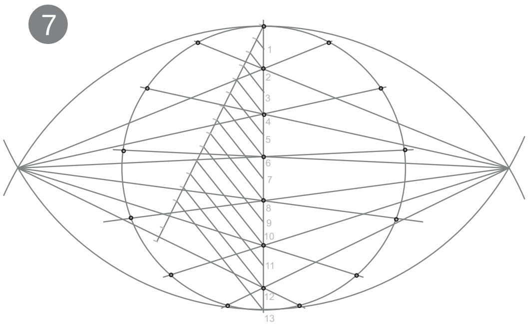


5th- Centered in the diameter's end points and radius equal to it, trace two arcs in which both intersections we will find two focii (one focus in each intersection).
6th- From one focus we trace rays through the even divisions on the diameter to intersect the circle in two points every ray. These rays project on the circle half of the divisions in their outgoing intersections with the circumference. Division 0, on the diameter also must be included, even though we didn't need to project a ray due to its position on the circle.

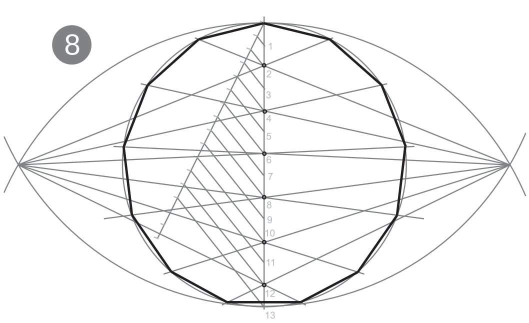




7th- Repeat the last step, this time in the opposite side.



8th- Conect all the points obtained on the circle. Remember to conect also number 0 on the top of the diameter.

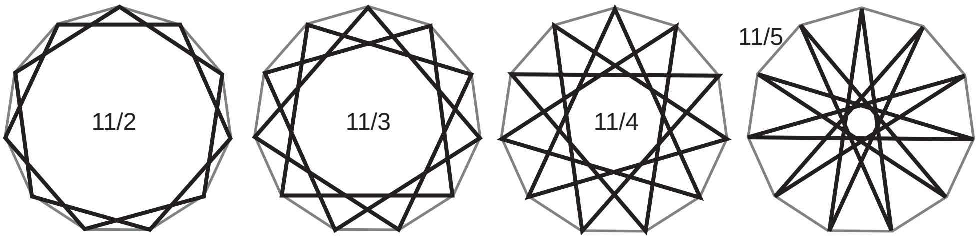


The star polygons are obtained by connecting continuously and non-consecutive vertices of regular polygons.
Depending on the number of vertices that the original regular polygon has we get none, one or more star polygons:

vertices	posible stars	way of joining vertices
5	1	2
6	0	-
7	2	2-3
8	1	3
9	2	2-4
10	2	3-4
11	4	2-3-4-5
12	1	5
13	5	2-3-4-5-6
14	4	3-4-5-6
15	4	2-4-6-7
...	...	...

To illustrate the box on the left we show the example of endec agon, which can hold up to four stars depending on the number of vertices we skip.

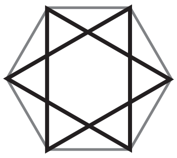
Conecting vertices skiping one to join with the second . | Conecting vertices skiping two to join with the third. | Conecting vertices skiping three to join with the forth. | Conecting vertices skiping four to join with the fith.



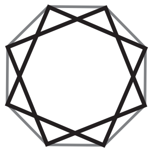
They are defined by N/M when N is the number of vertices in the regular convex polygon and M the number of vertices skipped. N/M has to be an irreductible fraction, other way the strar polygon cannot be formed.

In order to find out how many star polygons can be inscribed in a convex polygon:
n is the number of vertices in the regular convex polygon.
It is possible to make as many star polygons as integer prime numbers exist smaller than half of the number of sides or vertices (n/2) and prime numbers with n.
Example: Eptagon (7 sides), its half is 3,5 and the prime integer numbers smaller than 3,5 are 2 and three. Therefore we can join the vercices 2 by 2 or three by three.

STAR FIGURES, IMPROPER STAR POLYGONS OR COMPOUND POLYGONS



Star of David or hexagram.

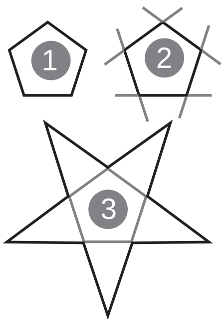


Improper star octogon.

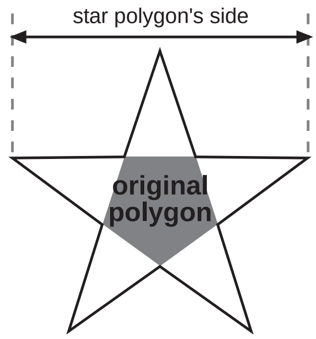
Sometimes, by connecting the vertices alternately we can find that actually other convex polygons are inscribed in the initial polygon. In such cases we will not get true star polygons.

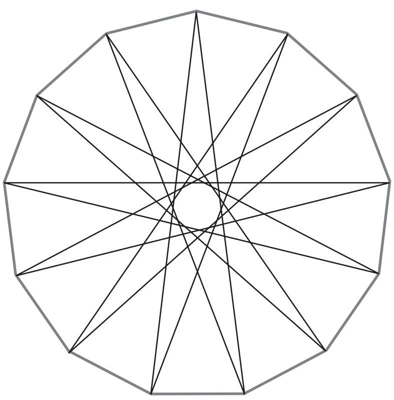
STELLATED POLYGONS

Stellating a polygon is to extend the sides to be cut back each other, so you get a new star-shaped polygon.

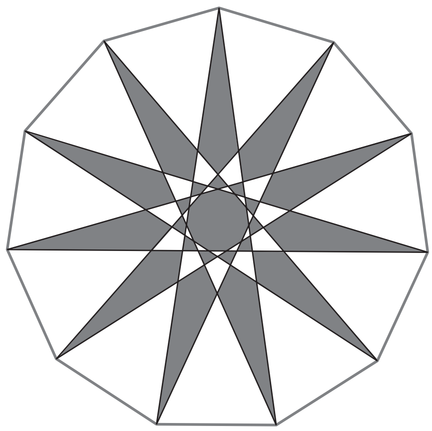


On the left we see the process of stellating a convex pentagon.
This polygon can be only stellated once, because the Pentagon only generates one star polygon.
The star Pentagon, also called Pentagram or Pentacle, is a very significant figure symbolically, especially to contain the golden ratio in their proportions





When stellating a convex polygon note that the first star that is generated is the one generated when smallest number of vertices are skipped to inscribe the star in the convex polygon. If we continue stellating it, we get the second smallest skiped vertices star. And so we draw, one inside the other, all the stars possible that the convex polygon can generate. The same applies we inscribe stars in a convex polygon starting by the highest number of skipped vertices (reverse procedure).



Basic concepts of geometry:

Segment: A portion of a straight line defined by two endpoints.
Straight Line: A line traced by a point travelling in a constant direction.
Perpendicular segment bisector: A line that intersects or meets a segment through its midpoint in two equal parts forming 90º angles.
Angle bisector: A straight line through the vertex of an angle which divides it in two equal angles.
Thales theorem: Two lines intersected by a set of parallel lines get divided in proportional portions. This theorem is used mainly to divide segments in equal parts.
Axis: A straight line that express a symmetry a height or width regarding a figure or geometric set of elements.
Perimeter:The path that surrounds a two-dimensional shape also referred as the boundaries.
Perpendicullar:They are lines which meet forming four right angles.
Parallel:They are lines which never intersect theirselves, so all their points are equidistant.

Polygons:

Polygon: A polygon is a flat and closed shape formed by straight sides joined by vertices.
Side: A segment which belongs to a polygon.
Vertex: A point in which two sides of a polygon converge.
Diagonal: The line segment connecting two nonadjacent vertices in a polygon
Star Polygon: They are the result of connecting non consecutive vertices of a polygon.
Pentagram: A star polygon.
Hexagram: A star hexagon.
Triangle:A three sided polygon.
Quadrilateral: A four sides polygon.
Square: A four equal sides polygon with four 90º vertices.
Rhombus: A four equal sides polygon with two couple of equal vertices.
Hexagon: A six sides polygon.
Heptagon: A seven sides polygon.
Enneagon:A nine sides polygon.
Octagon: An eight sides polygon.

Circle:

Circle:A set of points equally distant to a point called center.
Center: A point which is equally distant from any point of a circle or from any vertex of a regular polygon.
Circumference: the distance around the outside of a circle, that is to say, The circle's perimeter.
Radius:Distance between the center and any of the points of the circumference.
Diameter: A line through the center of a circle which conects two points of it.
Chord: Segment which joins two circle points not through its center.
Arc: A portion of a circle.