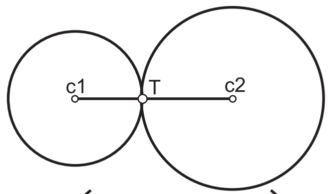
Tangencies

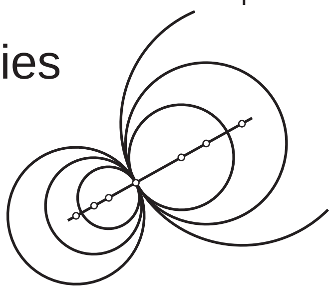
Two elements are tangent when they have a common point called the point of tangency. These elements are circles (or circle arcs, in some cases also conic curves) and straight lines.

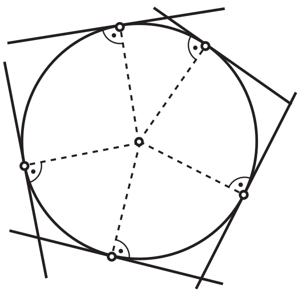
A link is the harmonious union point of curves with straight or curved corners. Links are the practical application of tangencies.

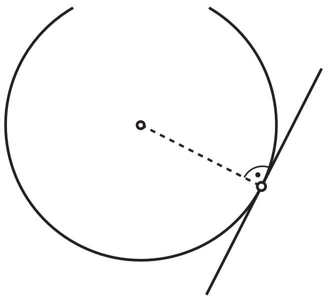
Fundamental properties of tangencies

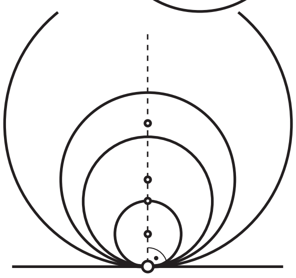
1- The centers of two circles tangent to each other are colinear with the point of tangency.





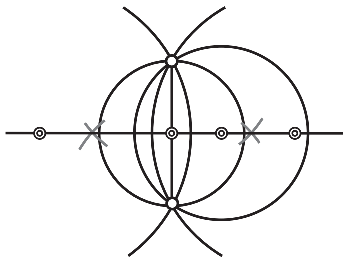


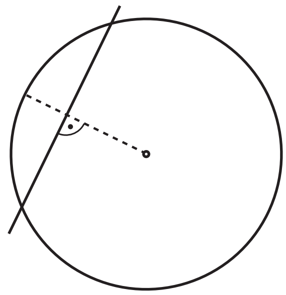


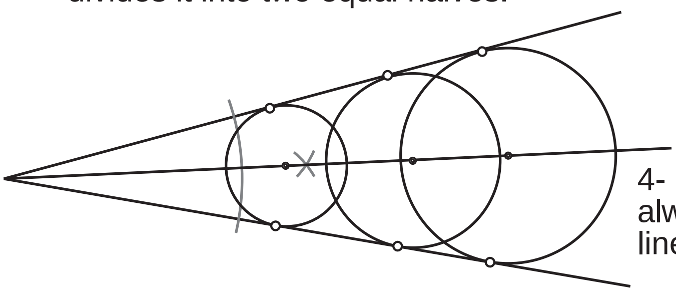


2- A line tangent to a circle is always perpendicular to the radius corresponding to the point of tangency.

3- The center of any circle through two points is on the bisector of the segment defined by those two points. Any radius perpendicular to a chord of a circle divides it into two equal halves.

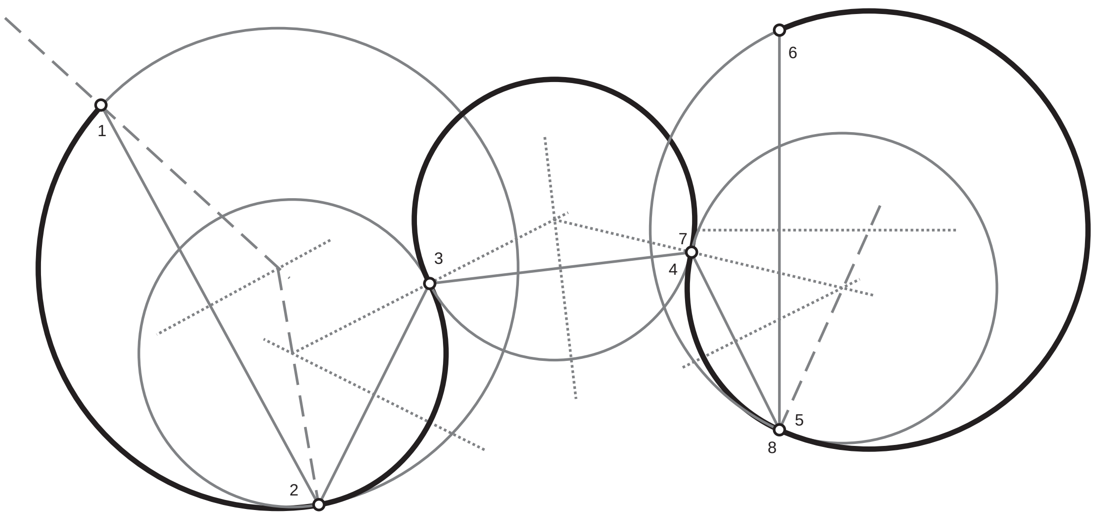






4- The center of any circle tangent to two lines is always in the bisector of the angle that the two tangent lines form.

JOINING POINTS WITH CIRCUMFERENCE ARCS TANGENT TO EACH OTHER



1st- The first arc is given with its center or we trace it with center on the perpendicular bisector of the two points that must be on the arc. THE CENTERS OF ARCS CONTAINING THE SEGMENT'S END POINTS ARE ALWAYS O THE PERPENDICULAR BISECTOR

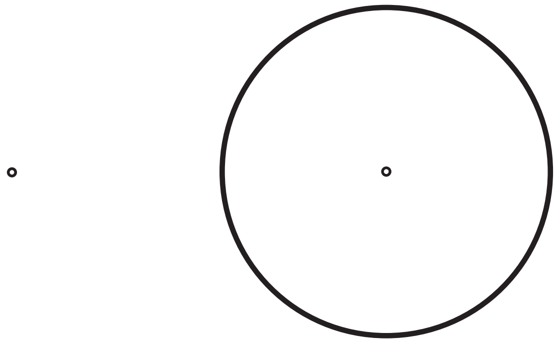
2nd-We can join the points with segments while tracing each arc or join all the points before tracing any arc.

3rd- Trace the perpendicular bisector to every segment. Conect the last end point of every arc with its center with a line. On the intersection of that line with the perpendicular bisector of the next segment we will find the next arc center.

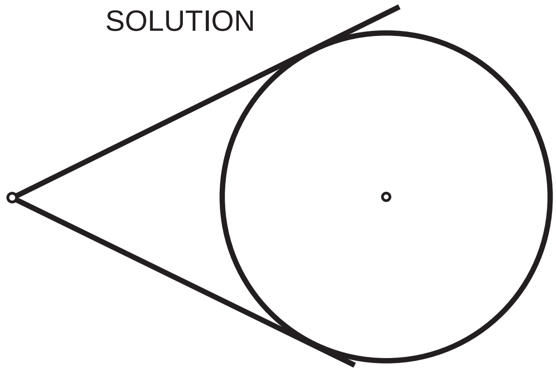
4th- Keep on taking the same procedure till we complete every segment end points conected or linked by arcs.

The graphic formulation shows a circumference, its center and an outer point. The problem will be solved making use of tangent straight lines through the outer point

GRAPHIC FORMULATION



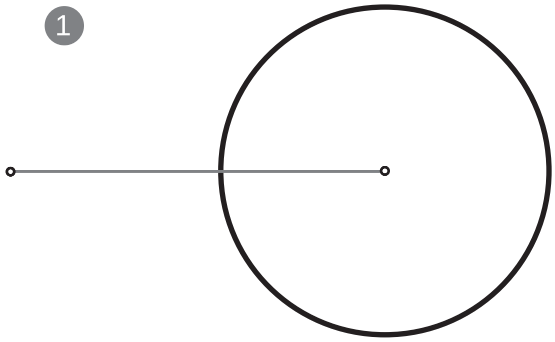
SOLUTION



In order to solve the problem we need some auxiliary paths that can be explained in four steps.

1st- Conect the circumference center with the outer point with a segment line.
2nd- Draw the segment's perpendicular bisector obtaining the segment's middle point.
3rd- With center on the middle point and a radius half of the segment trace a circumference that cuts the given one in two points, these are the tangency points.
4th- Trace the circumference radius to the tangency points.
5th- Trace two straight lines from the outer point to both tangency points.

1



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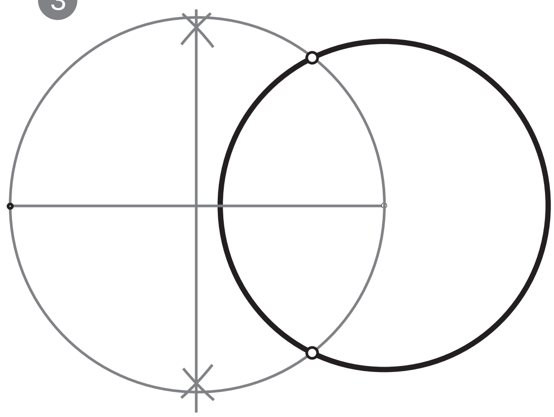

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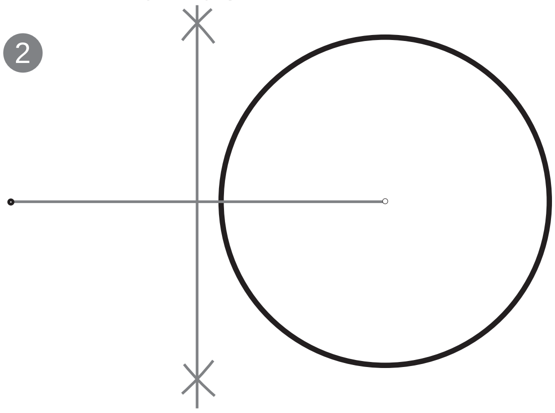

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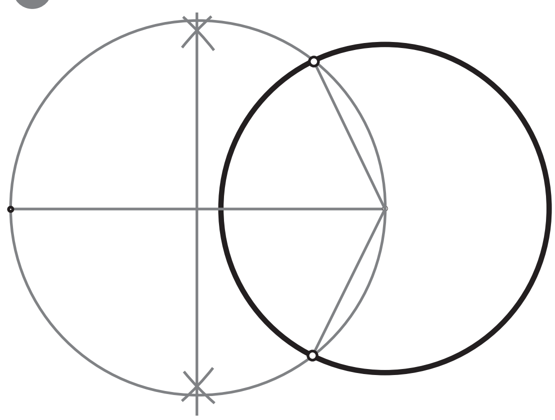

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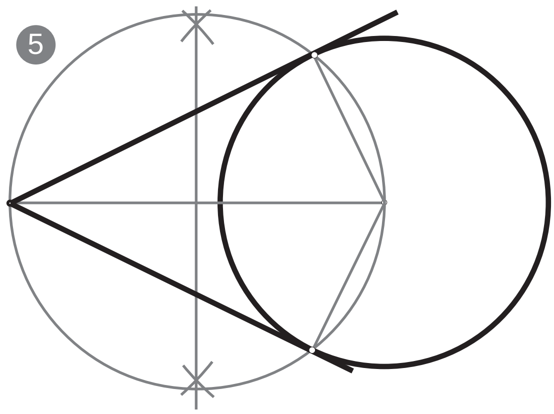
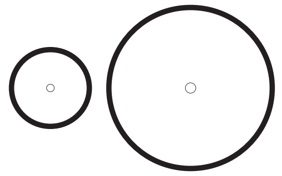
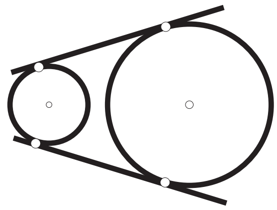


Inner and Outer tangent lines to two circles

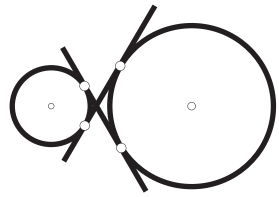
GRAPHIC FORMULATION



SOLUTION Outer tangent lines



SOLUTION Inner tangent lines

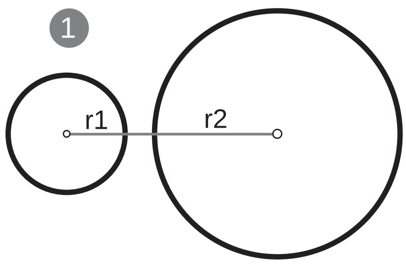


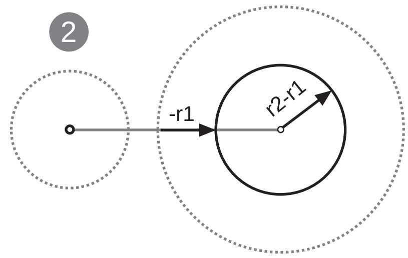
We need to reduce the problem to tangent lines to a circumference through a point in order to solve these problems. We need to make an effort to forget the initial graphic formulation. Once solved the reduced problem it is not too difficult to take the solution lines to the original formulation solving the initial problem.

Outer tangent lines to two circles

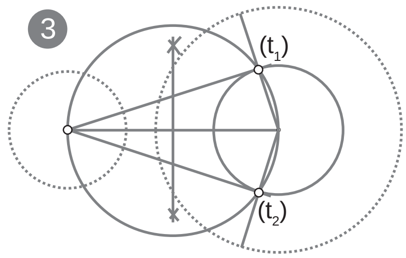
1th- Trace the segment joining both centers.
2th- On the segment, substract the smaller radius to the bigger one obtaining a smaller circumference inside the big one. The smaller given one turns into a point.

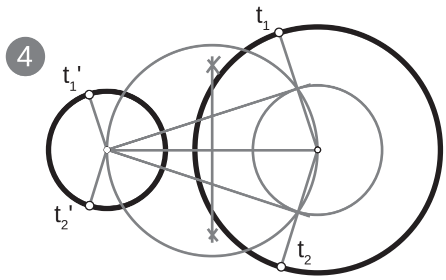
THIS WAY WE HAVE REDUCED THE PROBLEM TO TANGENT LINES TO A CIRCUMFERENCE THROUGH AN OUTER POINT

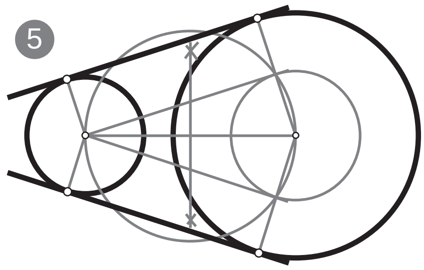




3rd- Solve the reduced problem, tracing radius to T1 and T2 long enough to cut the initial formulation circumference.
4th- Trace parallel radius through the smaller circumference center. So the four points obtained in the intersections of both initial circumferences are the tangency points.
5th- Join t1 with t1' and t2 with t2'



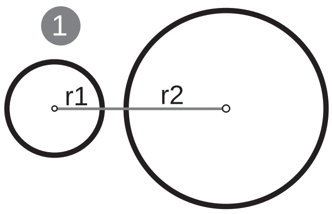


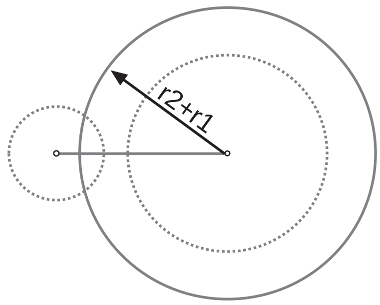


Inner tangent lines to two circles

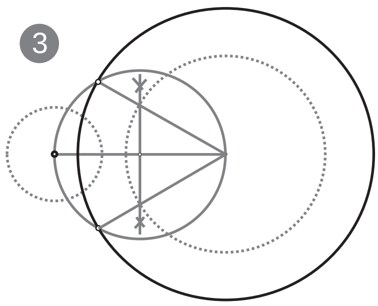
1th- Trace the segment conecting both centers.
2th- On the segment, add the smaller radius to the bigger one obtaining a bigger circle out of the big one. The smaller given one turns into a point.

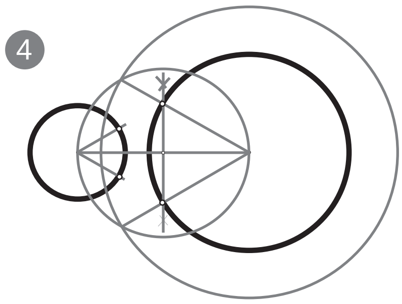
THIS WAY WE HAVE REDUCED THE PROBLEM TO TANGENT LINES TO A CIRCUMFERENCE THROUGH AN OUTER POINT

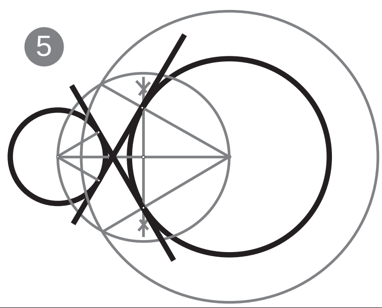




3rd- Solve the reduced problem, tracing radius to T1 and T2, these cut the initial formulation bigger circumference in two tangecy points part of the final solutions.
4th-From the center of the initial smaller circumference trace two radius parallel to the first pair traced but this time inverting the position directions upside down. The points where these radius cut the circumference are the other pair of tagency points wanted.
5th- Join t1' with t1 and t2' with t2

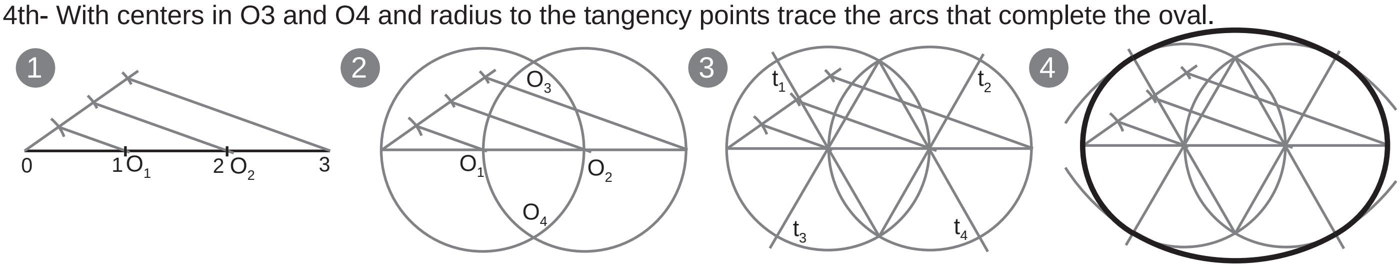






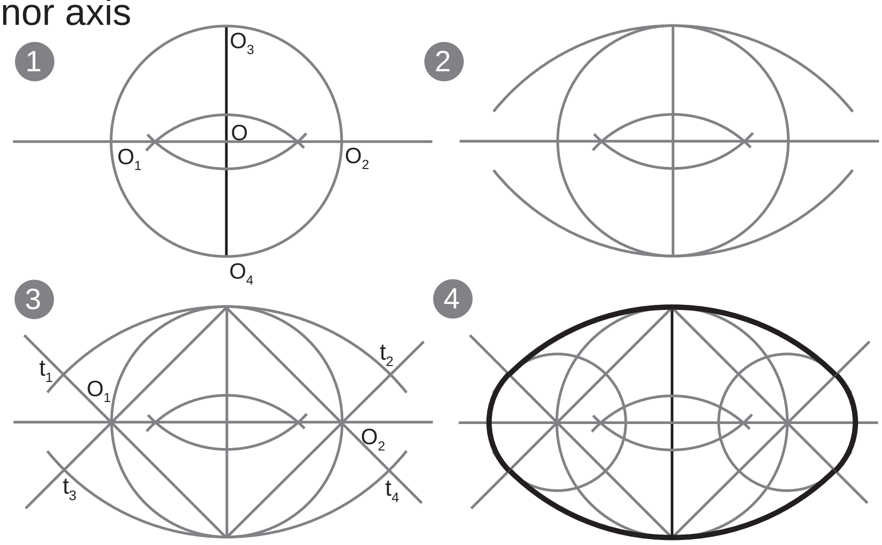
Oval construction given the major axis

1st- Divide the major axis into three equal portions. The two points that set the division are two centers for two of the oval's arcs
2nd- With center in O1 and O2 trace two circles with a radius equal to one third of the major axis.The intersections of both circles are O3 and O4
3rd-Conect O3 and O4 with O1 and O2, the points in which these lines meet both circumferences are the tangency points of both of them with the other two arcs
4th- With centers in O3 and O4 and radius to the tangency points trace the arcs that complete the oval.



Oval construction given the minor axis

1st- Display vertically the given axis, trace its perpendicular bisector and with center in the minor axis middle point trace a circle with a diameter equal to the given axis obtaining in both diameters the four centers for the oval's arcs
2nd- With centers in the minor axis end points trace two arcs with a radius equal to the full minor axis.
3rd- Conect O_3 and O_4 with O_1 and O_2 Obtaining over the arcs the tangency points.
4th- With center in O1 and O2 trace the necesary arcs to complete the oval with a radius open to the tangency points.

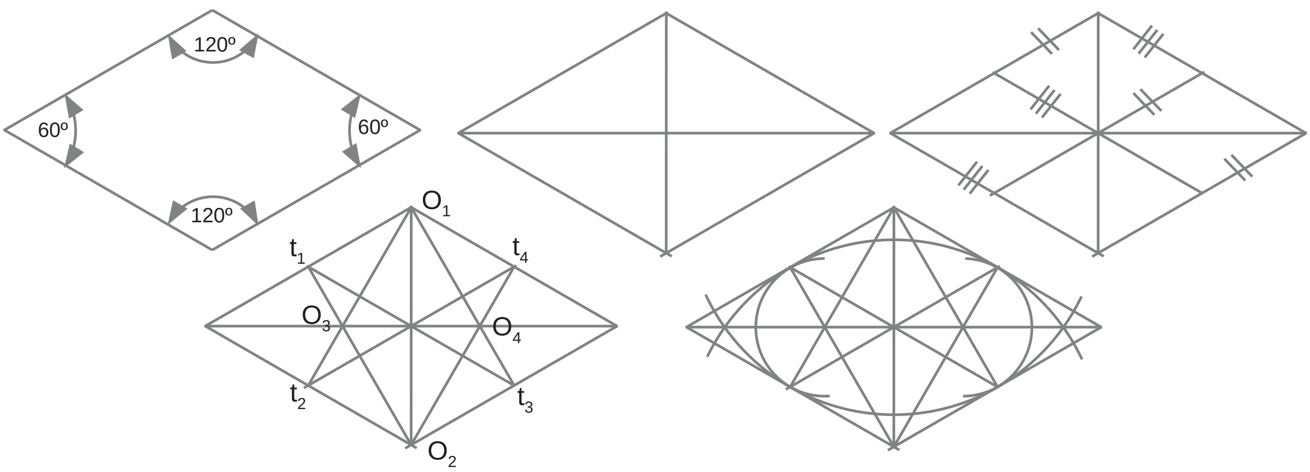


Oval construction given the circumscribed isometric box

Ovals are used to depict circumferences in axonometric systems. Actually any circle observed in perspective is an ellipse. But the ellipse can only be traced through determining points and that takes long. So with a purpose of a clean and clear depiction ovals are allowed in axonometrics to depict circles.

In axonometric system it is frequent to find boxes (flat or tridimensional) in which circles or volumetric shapes containing them need to be inscribed.

Below we can see how to trace an oval inscribed in an isometric box, that is to say a rhombus whose opposite angles are 120º and 60º.



Ovals are closed and flat curves formed by at least four symmetric arcs. It is usually defined by two axes that work as symmetry axes for the arcs. It is used frequently in axonometric systems to depict circumferences. When the oval has only one symmetry axis they are called **eggs**.

A **spiral** is an open and flat curve that turns around a point and away from it.

There are many kinds of **spirals**. Most of them cannot be **depicted** using a compass, those are the most common ones in **nature**. Spirals appear very often in nature mainly in **earth and sea snails**, but also in **animal horns**. Flowers, like the sunflower, where all the seeds in the middle seem to grow in spiralish directions. When spirals have **three dimensions** they are called helices; tornados are conic **helices**. Also the **galaxies** look like spirals.

Martín Chirino: 1976
El viento (50) El viento de Canarias.

Spirals were always used by **artists** for many abstract **depictions**. Spirals have a **symbolic sense** that has been used since the **prehistoric age**, very ancient painted spirals have been found **painted or curved on stones** in all five continents.

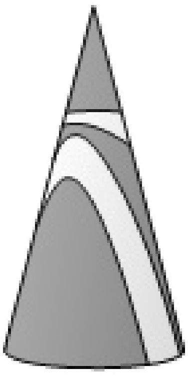
These **curves** are used in design for **decorating** any kind of **objects** or **architectures**. An ancient type of greek style, called **ionic**, used **volutes**, a couple of stone carved spirals, on the top of its **columns**. Also Martin Chirino, who is a Spanish **sculptor** is well known for **drawing** and **forging** iron to create different kinds of spirals. Sonja Hinrichsen is a current artist who walks on the snow drawing spirals with her **footprints** while she walks.

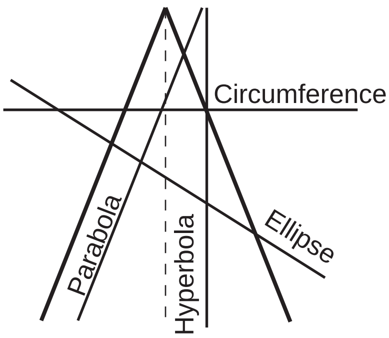
Sonja Hinrichsen

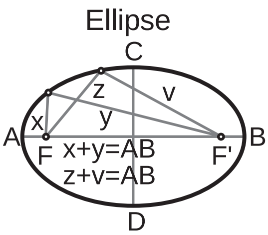
- Why do you think spirals are so frequent in art, architecture and design?

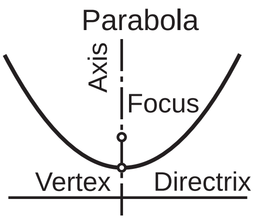
- What do you think an spiral can stand for, what things could you say a spiral symbolize?

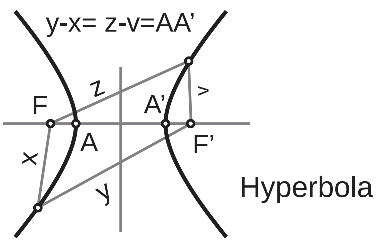
- For what kind of feelings or emotions would you use an spiral for a drawing?











APOLLONIUS CONE

Apollonius of Perga was a Greek mathematician who lived 200 years before Christ. Known for his books about **tangencies,** he became more famous because of his eight books about the **conic curves**.

In the fourth book about conics, Apollonius explains the ways in which **a plane can cut a cone to produce the four conic curves**. So a wood cone with five removable parts describing the four curves is known as the **Apollonius cone**.

Depending on how a plane intersects the cone the section produced can be a **circumference, parabola, ellipse or hyperbola**.
These curves have **many applications** in science and study of our environment in the Earth and the Universe.

The planets describe **elliptic orbits** around the sun, the **circles in perspective are seen as ellipses**.
Any projectile describes a **parabolic path**. And also a beam of parallel rays reflected on a parabolic surface is concentrated at a point which is the focus, and operate the **satellite dishes**, a type of antennas.

Comets describe **hyperbolic paths** and **sundials** are based on the knowledge of the hyperbola.

These are just some examples of the presence of these curves in our world. Science has helped us a lot through this knowledge brought by Apollonius.

Tangencies:

Inscribed: Figure within another, whose vertices also belong to the outer figure, but not intersecting to the outer one.
Circumscribed: Figure drawn around another geometric figure touching it at all its vertices but without intersecting it.
Tangent: A line which shares a point in common with a curve, but it doesn't intersect it.
Point of tangency: A point which joins together two elements such as straight lines or curves.
Link:The practical or handy expression of point of tangency.

Other curves:

Spiral: a curve line which grows in a orderly way from and around one or several centers or a core.
Oval: A curve formed by four or more linked arcs. They are defined by a major and a minor segments which perform as symmetry axes.
Egg: They are a particular case of ovals with only one symmetry axis.
Conic curve: A curve produced by the intersection of a plane with a cone. There are three types depending on how the plane intersects the cone.
Ellipse: A conic curve whose addition of distances from any of its points to both focci (pair of characteristic points on the major axis) is always the same.
Parabola: A conic curve where every point on the curve is equidistant from one point (the focus) and a line (the directrix)
Hyperbola: A hyperbola is the set of points where the difference of the distances between two points is constant.