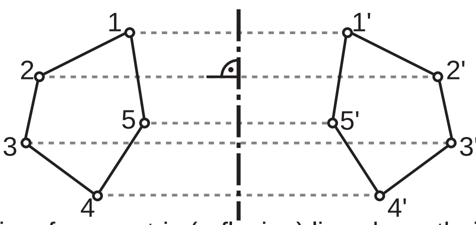
PLANAR GEOMETRIC TRANSFORMATIONS: SYMMETRY ROTATIONS, TRASLATIONS AND DILATION.

ISOMETRIC (= legth)	The original figure and the transformed one keep the same lengths in sides and angles.	ROTATION TRASLATION SYMMETRY
ISOMORFIC (= shape)	They keep the same shape but different size.	DILATION
ANAMORPHIC	Figures change the size and also the angles lengths.	INVERSION HOMOLOGY AFFINITY

SYMMETRY

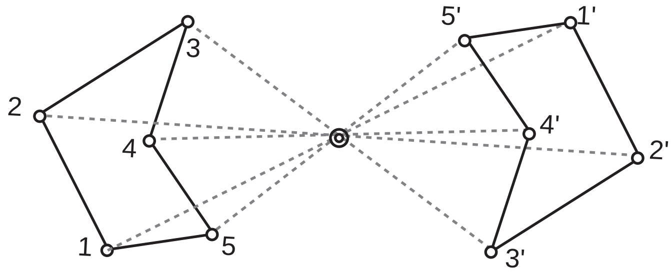
SYMMETRY:It is a geometric transformation in which every point and its simmetric are in the opposite side of an axis or a center and at the same distance from it. There are two types of symmetry:

REFLECTION SYMMETRY (axis-line): Symmetric points are on a perpendicular line to the symmetry axis, at the same distance from it and in opposite sides of it.



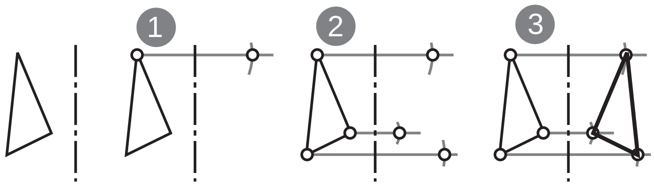
Pairs of symmetric (reflexion) lines have their intersection on the symmetry axis. When the axis meets a straight line, its simmetric line cuts the first one on the symmetry axis. The intersection point of these symmetric lines is called DOUBLE POINT. Any point over the symmetry axis has its symmetric on the same point, that is called DOUBLE POINTS.

POINT SYMMETRY (center-point): The simmetric points are alligned with the center of symmetry, at the same distance and in opposite side of it.



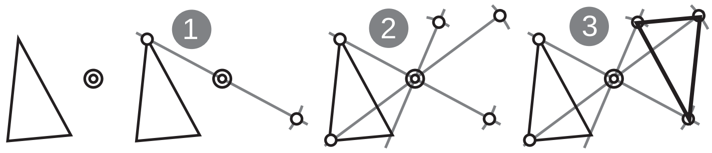
Central simmetry is the same as a 180º rotation with the same center. It is also equivalent to two reflexions with interseting perpendicular axes over the symmetry center.Simmetric lines or segments around a center are parallels.

Traze the symmetric triangle about the given axis.



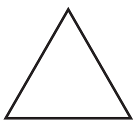
1.- Traze the perpendicular line to the symmetry axis through one of the vertices. With the compass we copy the distance from the vertex to the axis on the other side to obtain the symmetric vertex.
2.- Repeat this step with every vertex.
3.- Conect the symmetric vertices.

Traze the symmetric triangle about the given symmetry point.

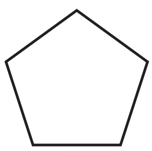


1.- From one of the vertices trace a line through the symmetry poynt. Make center with the compass on the symmetry point and copy the distance from the vertex to the center on the other side to obtain the symmetric vertex.
2.- Repeat this step with every vertex.
3.- Conect the symmetric vertices.

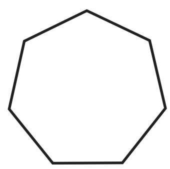
ROTATIONAL SYMMETRY takes place when a figure can be rotated around a center and mantains the same shape. Therefore the regular polygons have rotational symmetry or fold symmetry with a number related with the number of sides.



3 fold simmetry



5 fold simmetry

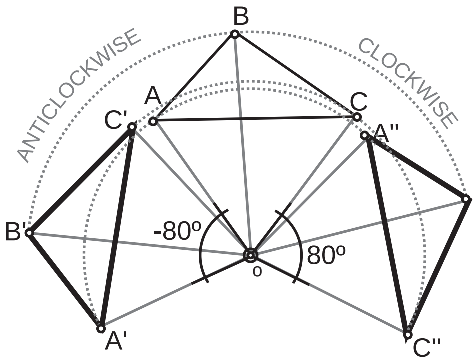


7 fold simmetry

ROTATION

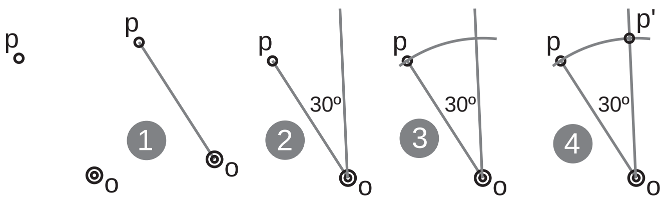
It is a geometric transformation in which there are: a center, a rotation angle and a direction.

The direction can be clockwise, in that case the rotation would mean a positive angle, or anticlockwise (counterclockwise in the US) being that way a negative angle rotation.



ROTATE THE POINT(p) AROUND A CENTER (o):

- Rotate the point p 30 º around the center o.



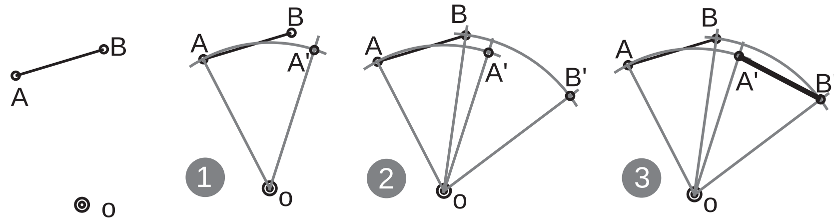
1.-Trace the segment OP.
2.- With the vertex on o, with the 30º triangle ruler or with a protractor trace another segment forming an angle with 30º.
3.- With center in o and a radius OP trace an arc that meets the last traced segment.
4.- In the intersection of the arc and the second segment we find the point p' as the result of turning (or rotating) p 30º clockwise.

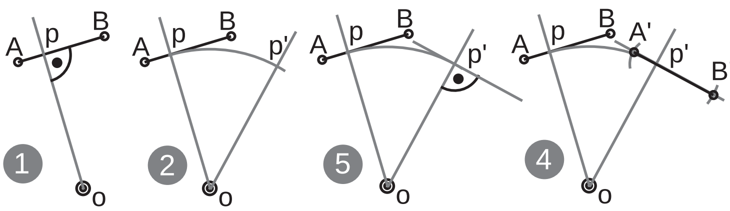
ROTATE THE SEGMENT *AB) AROUND A CENTER (o):

- Rotate the AB segment 45 º clockwise around the center o:

Rotating both endpoints:

1.- Using the method above, rotate the segment A end point.
2.- Rotate B end point the same way.
3.- Conect A' with B'.



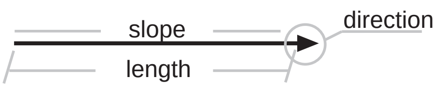


Tracing a perpendicular line to the segment:

1.- Trace a perpendicular line to the AB segment through the rotation center obtaining a point p on it.
2.- Turn (rotate) p, obtaining p'
3.- Trace a perpendicular to the radius through p'.
4.- Over this perpendicular, from p', copy the distances pA and pB. Trace the segment which is the result of the problem.

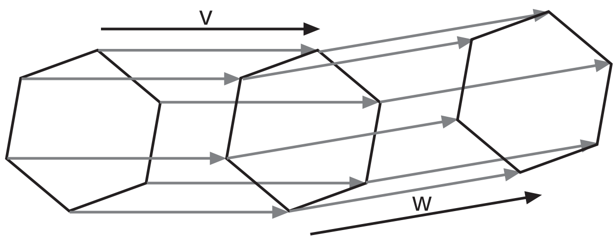
TRANSLATION

It is a geometric transformation or a push on the plane determined by a translation vector. A translation vector is determined by a length, slope and direction.



A translation motion can be defined by:

1- A figure and a translation vector.
2- A pair of points(original and translated).

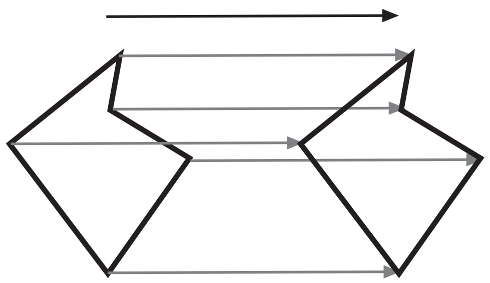


It is as simple as tracing parallels to the vector slope and in the given direction by the arrow from the figure's vertices, copying the length with the compas, to obtain the translated figure.

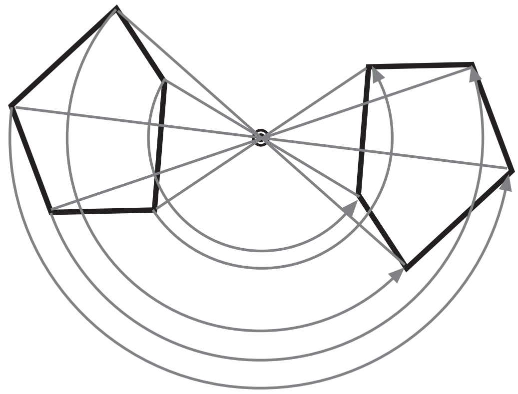
GEOMETRIC RELATIONSHIPS: EQUALITY, SIMILARITY AND EQUIVALENCE.

EQUALITY: In geometry two figures are equal when they have the same shape and size (so they both have the same area as well).

TRASLATION: Below. A pentagonal shape has been translated regarding a translation vector while keeping its shape and lengths.

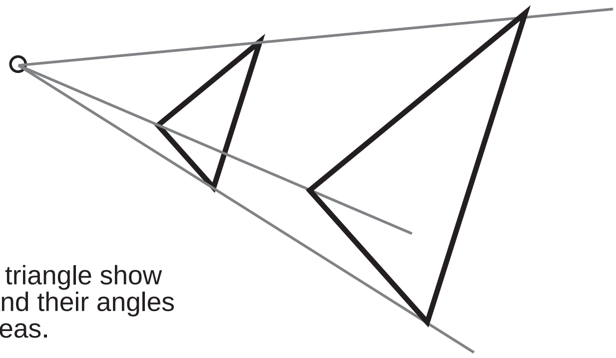


ROTATION: Below. A pentagonal shape has been rotated 180º in counterclockwise (anticlockwise) direction while keeping its shape and lengths.



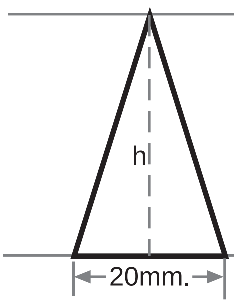
SIMILARITY: Two figures are similar when they have the same shape but different size and so different areas.

Similar figures keep the orientation and angular lengths, but they are different in their sides lengths attending to a proportion ratio, that is to say, two similar figures are proportional.

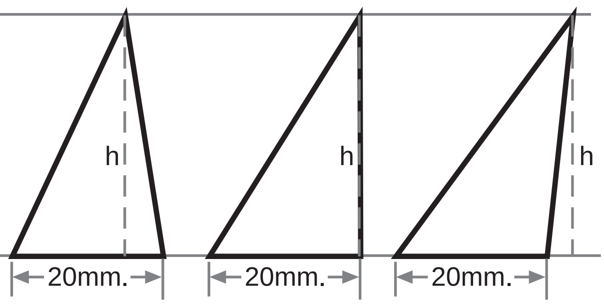


Dilation: On the right. A triangle and its dilated triangle show the same shape, their sides are proportional and their angles are equal, but they have different sizes and areas.

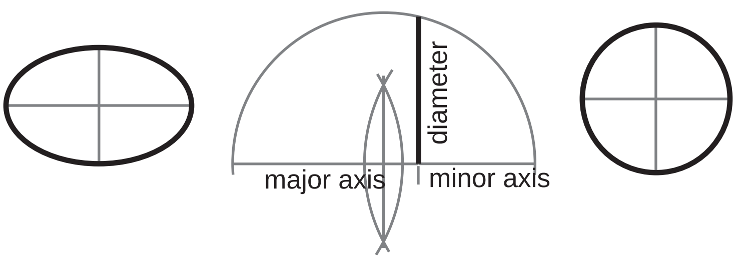
EQUIVALENCE: Two figures are equivalent when they have different shapes (or not) but they have the same area.



The area for an isoceles triangle is given by the formula =(b·h)/2.. So all the triangles on its right are equivalent because the keep the same base (20mm.) and the same height.



EQUIVALENCE ELIPSE - CIRCUMFERENCE



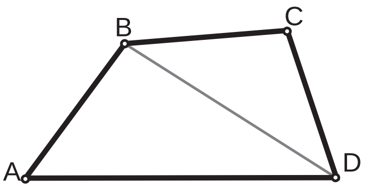
"halved axes of an ellipse are average proportion with the radius of the equivalent circumference".
In other words:
" the axes of an ellipse are average proportion with the equivalent circumference's diameter".

EQUALITY

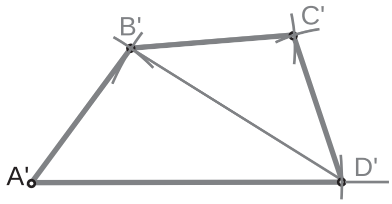
Two figures are equal when they keep the same shape and the same size. Two equal figures always have the same area. For polygons applies equality: same angular magnitudes at the vertices, same magnitudes of the sides and therefore equal area.

GIVE THE ABCD QUADRILATERAL, COPY IT FROM A': by triangulation

Any polygon with more than three sides can be decomposed into triangles. For this, we can decompose the polygon we want to copy into triangles and copy them one by one. This way we avoid using the angles copying procedure which is somehow imprecise if we are not careful and we can copy the polygon using only the sides of the triangles.

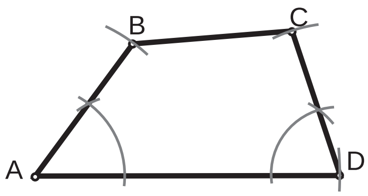


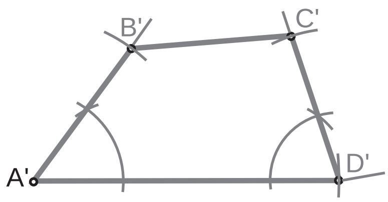
First copy the triangle ABD from A '. Once done copy the BCD triangle on the side B'C '



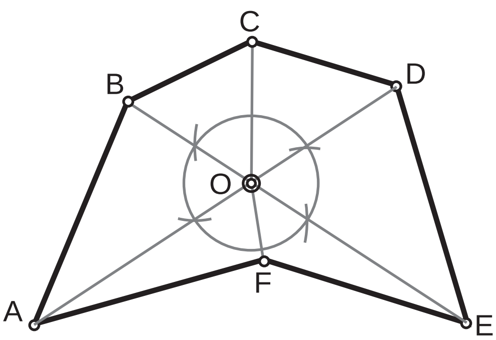
GIVEN THE ABCD QUADRILATERAL, COPY IT FROM A ': By angles and segments copying.

We simply use the angles and segments copying procedures to copy the polygon from the given point.

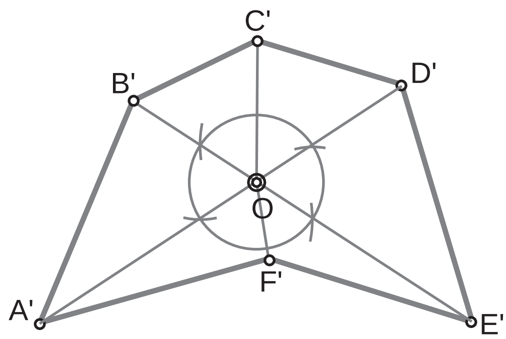




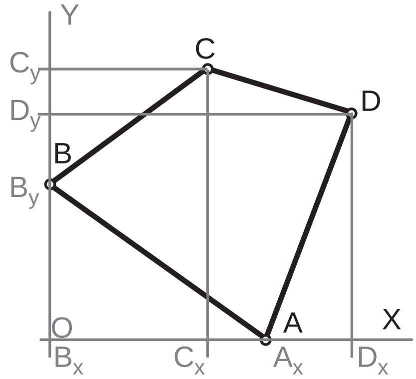
GIVEN IRREGULAR HEXAGON ABCDEF, COPY IT FROM A ': By radiation



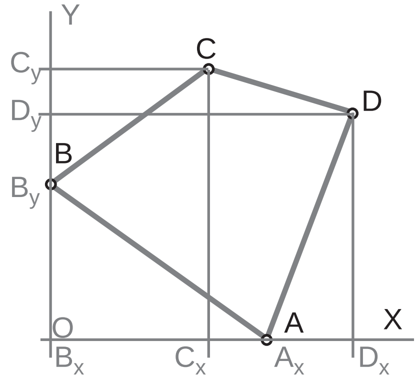
In this case it is about positioning a center from which the radii (radiuses) are drawn up to the polygon vertices.
With it we will draw another center and will copy the angle magnitudes between the radii in order to copy the distances between the center and vertices.
Notice how we only draw one circle to copy angular magnitudes, this circle must have the same radius in the formulation and in the result.



GIVEN THE ABCDE QUADRILATERAL, COPY IT FROM O ': By Coordinates



It consists of drawing two coordinate axes. These should form an angle of 90 degrees and if we position two vertices of the polygon with the axes we will save a step.
We will project ortografically the polygon vertices on each coordinate axis and then copy magnitudes of the segments in order to build again the polygon.

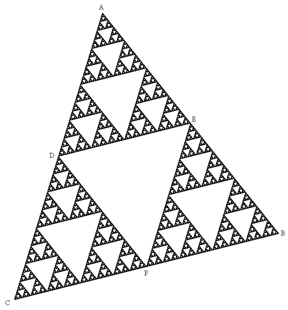


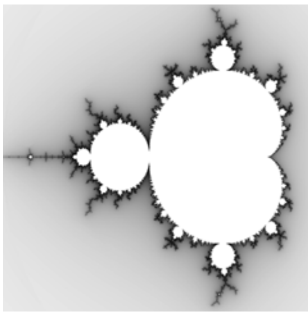
FRACTALS

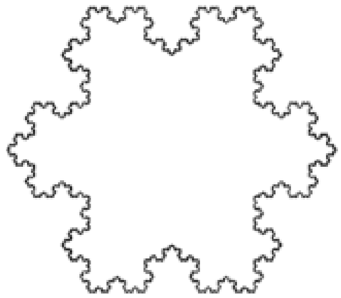
Fractal geometry was discovered in the 20th century . Fractals are mathematic formulas or equations whose points **describe** very **natural looking** shapes and which actually contain their own shape but repeated in different gradually smaller sizes. This quality of fractal shapes, also in many natural shapes, is called **self-similarity**.

In the second half of the same century, **Benoit Mandelbrot** who was a Polish mathematician set the theory about the **fractal shapes**. Besides his studies, Mandelbrot also drew the **Mandelbrot set**, that looks somehow like a black rough heart colored in the outside with bright colors. Other known earlier and simpler fractal is the **Sierpinsky Triangle** consisting in a black triangle filled in with smaller triangles smaller on and on. The **Koch snow flak**e is another simple fractal which is a triangle derived shape that looks like a scheme of how water gets frozen creating a kind of hexagonal star. The **Barnsley fern** is mathematic function whose points describe very accurately how a fern (a plant) looks like.

Attending to the descriptions above give the correct name to every image shown below







CHANGING PROPORTIONS IN SCULPTURE

It is very common that two visual dimensions depictions show the sizes of the objects smaller or bigger than the real objects of the representation. But in sculpture it isn't very common to see sculputures or artists who work changing the sizes of what they depict.

Check out **laslaminas.es Pinterest Fractals Gallery** by clicking on the link or scanning the QR code.

Charles simonds: He is an American artist well known by his "**Dwellings**" which were little **tiny constructions** he made with tweezers, sometimes **round sculptures** and other **built in holes** on the walls.

Claes oldenburg is a Swedish artist, pioneer of **pop art**, whose **art installations** or sculptures were **huge everyday objects**.

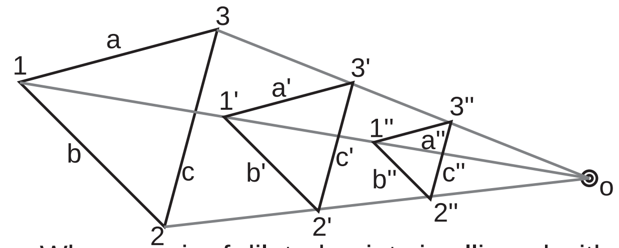
Takanori Aiba is a Japanese artist, architect, designer and civil engineer whose sculptures are scale models of imaginary or inspired buildings which are half way between sculputure and architectures of curious buildings or little towns.

Ron muek:He is an Australian hyperrealist sculptor living in England whose main feature is changing the **scale** of **human figure enlarging** or **reducing** the sizes of the people he represents.

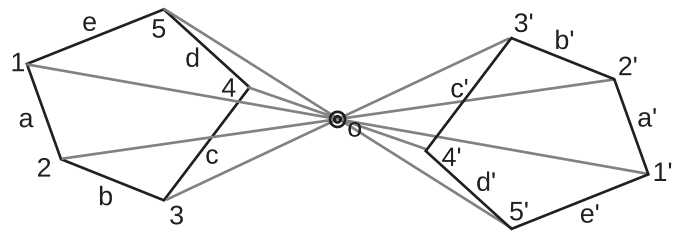
DILATION

Dilation is a geometric transformation, it is a geometric correspondence between two figures in which pairs of dilated points are alligned with the dilation center and dilated pairs of segments are always parallels.

DIRECT DILATION: SIMILARITY



INVERSE DILATION: CENTRAL SIMETRY + SIMILARITY



When a pair of dilated points is alligned with the dilation center and at the opposite side from it on the dilation radiation, this dilation is called INVERSE. When the pair of dilated points is at the same side from the dilation center on the radiations it is called DIRECT DILATION.

DIRECT DILATION: DIRECT dilated figures are similar and never equivalent. The scale factor for direct dilated figures is always positive.
INVERSE DILATION: INVERSE dilated figures attend to a negative proportion ratio, they are equivalent if the scale factor is -1. In this case figures are not similar but symmetric regarding two reflexion axes that intersect perpendicularly through the dilation center.

GRAPHIC SCALES

The **scale** is the **ratio**, usually expressed as a fraction, **between the size of the graphic or drawing (D) and the actual dimensions of the object (R)**.

D/R: drawing measures divided by the actual or real measures.

Reduction Scales: 1/2 (1cm from the drawing correspond to 2cm from reality. "Half of ..."), 1/5 (one fifth of ...). They are mainly applied in geodesy, topography and architecture.

Enlargement Scales: 2/1 (2cm from the drawing correspond to 1cm from reality). "Double of...", 3/2 (3cm from the drawing correspond to 2cm from reality).They are mainly applied in industrial design Plans, such as a nut.

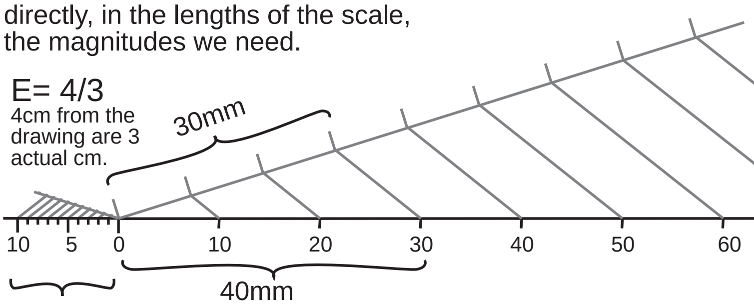
Full scale: 1/1 (drawing and real object measure the same). Whenever possible we choose this scale for the drawing.

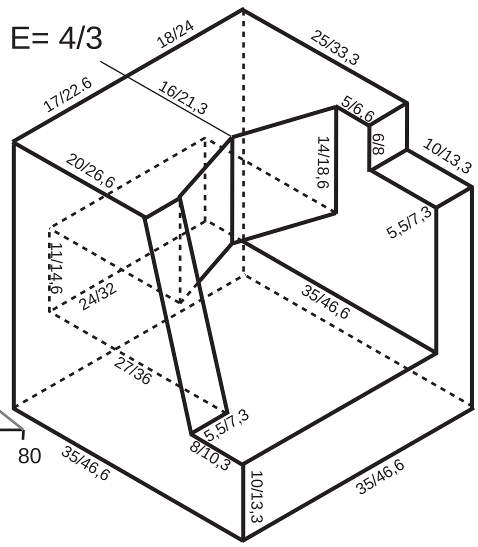
In any case the ideal scale always tries to find a balanced solution where you can clearly see every detail of the drawing. The selected scale is always conditioned by the object sizes and the format dimensions (A3 or A4 are more standardized) used for drawing.

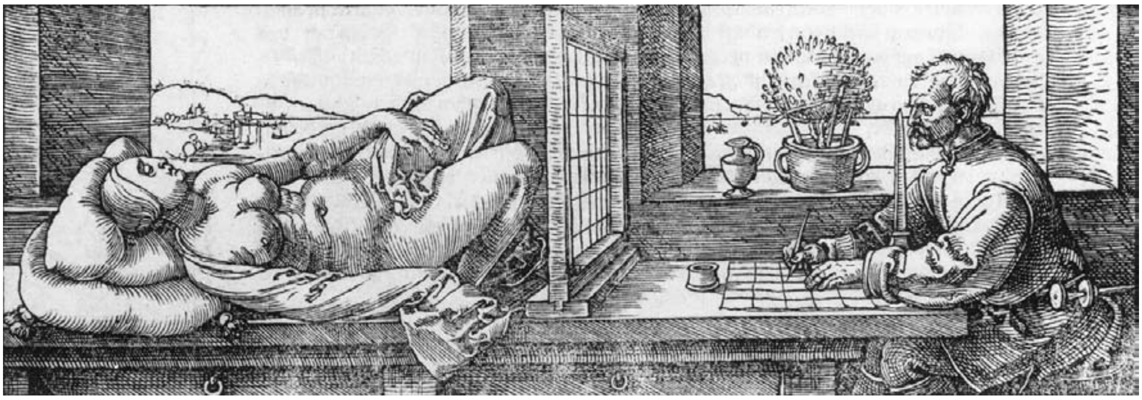
GRAPHICAL METHOD

Once the scale is defined we could make a note on the figure sketch or draw the measures that we will use for subsequent drawing by applying a multiplication and / or division. But this method is not really practical. Specially for parts or drawings in which we could find lots of different measures.

The construction of the scale will allow us to read directly, in the lengths of the scale, the magnitudes we need.







ALBERTI'S GRID AND ANAMORPHOSI

Fill in the blanks with the given words below:

During the **reinascence**, centuries 14th and 15th, it was investigated how to get the most **realistic depictions**. Some _________ and artists worked hard on discovering how to draw as accurately as possible in a similar way to human vision. Leon Battista Alberti was a priest during the Reinsacence who studied ______ and architecture. He thought of a way to draw consisting in a frame with some tight strings attached to it divinding it in squares.

The artist could look through it and ______ what he saw in each square onto the paper that had, previously prepared, a **similar squared grid** drawn. So copying the scene or object, **square by square**, is much easier than copying the full ________. This is a good technique to copy accurately any image, because the squares grid helps to place the elements in the right __________ and size. This instrument is called **Alberti's veil**.

Albert Albrecht Dürer was a German artist who, some years later, made ________ ilustrating the Alberti's veil way of drawing and some other drawing machines.

Also in Renaiscence **Anamorphosi** was used to encript messages or images that the artist didn't want to _______ in an explicit way. Some **anamorphosi are made to observe the drawing from a specific point of view** so perspective helps to recover the ___________ from that point of view letting the viewer see what it has been depicted clearly.

An easy way to create anamorphosi is using Alberti's veil, but this time distorting the ________ along with the perspective. Currently there are a lot of street artists who like to make anamorphosi in the streets, **Julian Beever** is one of the most famous. If we observe all his chalk walk paints we can notice most of the floors he uses tiles forming squares that help him to "translate" the drawing into an anamorphosi.

woodcuts · copy · show · image · position · grid · distortion · architects · arts ·

Scann this QR or click the link to learn some more about anamorphosis in **www.anamorphosis.com**

A **frieze** is a sculpted or painted **horizontal band**, sometimes done with geometric abstract elements some other with figurative images, which always uses translation to repeat the elements.

Depending on the type of frieze they may also show **rotation**, and **simmetry** or **reflexion**. They also could show another geometric transformation called **glide reflexion**, which is the product of a reflexion and a translation. A perfect glide reflexion example could be the walking **footprints** on the ground.

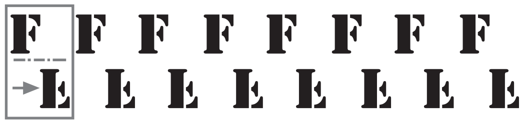
Believe it or not, there are only seven types of friezes atending to the types of **geometric transformations** they use.

Translations,vertical simmetry and glide reflections;

Translation and vertical reflexions frieze.

Translations and horizontal reflexions frieze.

Translations and glide reflexions frieze.

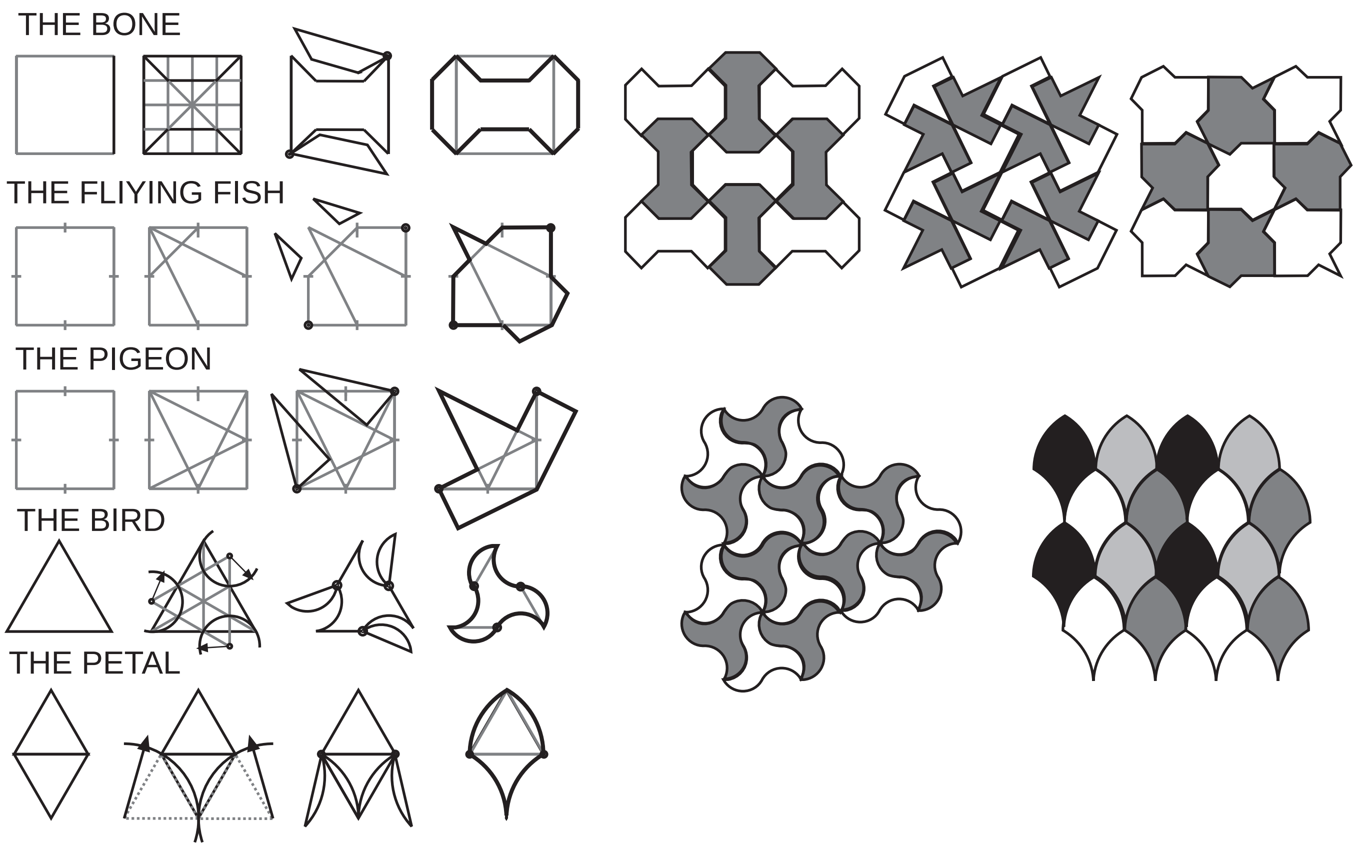


Translations and 180º rotations frieze.;

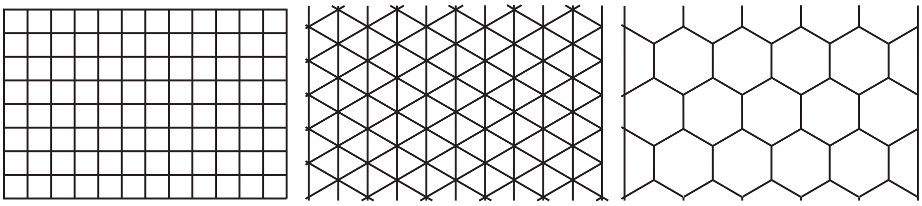
Translations frieze.

Translations, 180º rotations and horizontal simmetries frieze.

"La Alhambra" is a palace and fortress complex built in the 14th century. It is a good example on how muslims would **build** or **decorate**. La Alhambra shows the 17 kinds of geometric **tilings** and they did it in a very original way. Starting by **geometric patterns** whose tiles were basic polygons, they would change the polygons shapes into different figures having them **filling in the plane without leaving any spots left**.

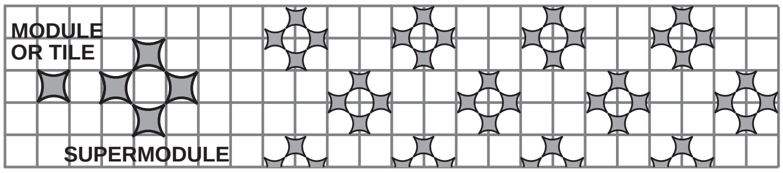


Modular grids: they are generally geometric structures in which a figure is repeated to form a composition. These figures are usually polygons or equivalent shapes.
Modular grids composed of figures that fill in the plane without gaps are called **tessellations**. There are only three regular tessellations (made repeating regular polygons).



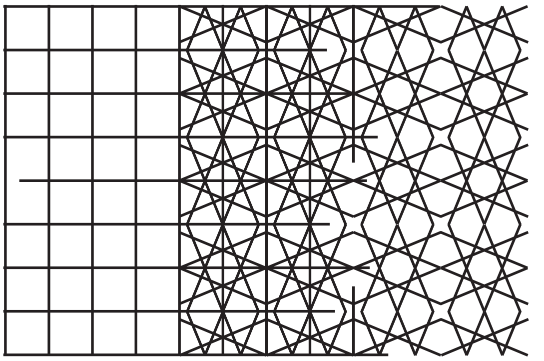
The **module or tile** is the basic figure which is repeated in the compositions of Modular grids. As shown in the drawings on the left, there are only three regular polygons that tessellate the plane, the **Regular grids**.

The **super-module** or **main module** is a figure composed of several basic modules which also acts as a module in a composition.



The Arabs were specialists in developing this type of decoration. In Muslim culture, because of the doctrines of the Koran, the artists and craftsmen must not represent human figures or animals in temples, religious objects or books. That is why they chose this way of decoration, in which modules are not recognizable figures of people or animals.

But Muslim culture is not the only one who has developed the partition of the plane. Mathematicians, artists and designers have also approached to study this interesting fact. **Escher** or **Vassarelly** are two very good examples.



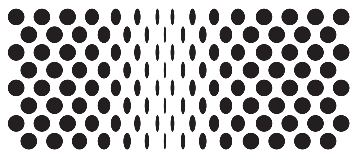
Simple Modular Network — Composite Modular Network — Simple polygon Network

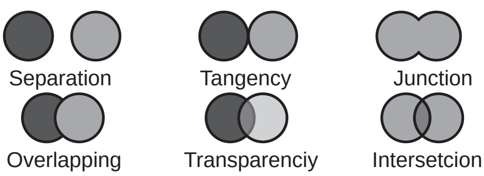
Simple grids are composed of one single figure repeated.

Composite grids: Are those consisting of two or more figures that repeat. When these figures are tilings must be polygons, even if they have different number of sides they must have equal sides.

There are also modules or **Modular grids** composed of **overlapping** networks or **simple tiles**.

The **anomaly** is a plastic resource that changes the order, the position or shape of the tiles or units to attract attention creating motion effects, or three-dimensional plane distortion.
Bridget Riley and other artists of the **Op Art** were experts applying this visual appeal.

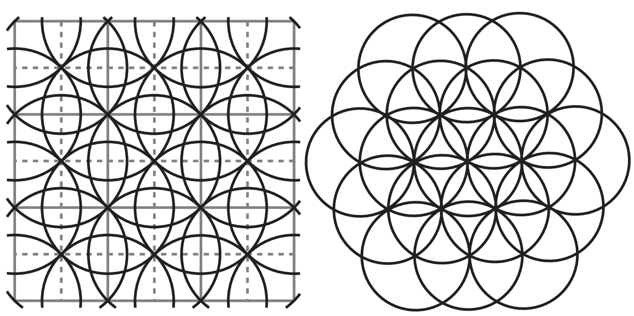




The circles are also very common in grids. But as they do not show sides in their outlines can not fill the plane in a tessellation. On the left, the ways in which the circles can be arranged to accomplish pattern compositions with them as tiles.

On the right we see two different ways to arrange the circles on the plane.
These two forms were the basis that Muslims used to form them, linking different intersections getting semiregular tessellations.
A **semi-regular tessellation** is one that with regular polygons (all with the same side length) while filling the plane leaving no gaps.

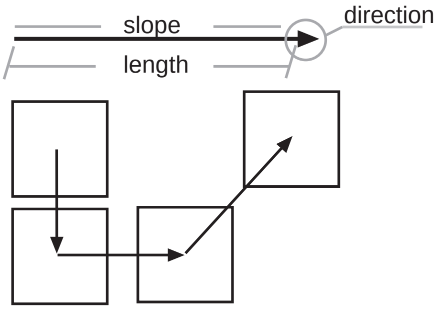
See how the muslims created designs for their tiles starting bycircles patters and other related videos in the same playlist scanning this QR code or clicking on the following link:
http://www.youtube.com/watch?v=kfSjfYhmqcM&feature=share&list=PLF1A8C4B7B614EF24

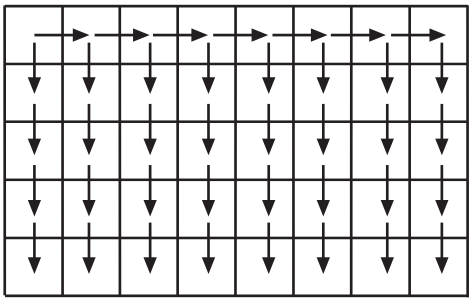


Movements in a plane:Dynamic Geometry: Isometries

A movement is transforming the position of a figure in the plane, in this case our cells or tiles. Specifically, when we apply a movement, the tile will hold its shape (its sides, its size, its area and its angles are equal: Isometry) but change its position on the plane. There are three types of Isometric movements:

TRANSLATION:

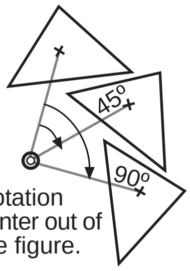




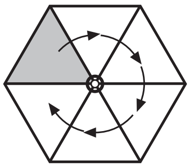
Translate a figure is moving it, pushing it. All translations are determined by a **vector**. A vector is determined by a **lenght** (modulus or distance), a **slope** and a **direction**

ROTATION O TURN

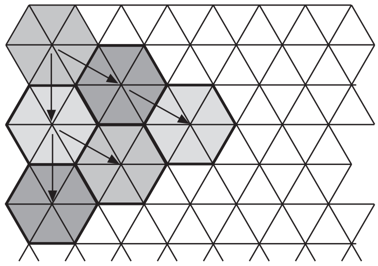
To rotate a figure you need a **center of rotation**, an **angular length** and a direction (**clockwise or counterclockwise**). The rotation center may be positioned inside or outside the edges of the figure



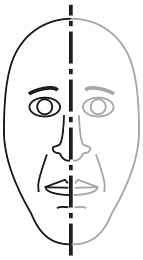
Rotation center out of the figure.

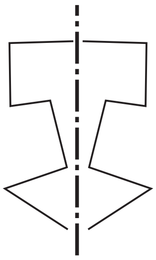


Rotation center on a vertex of the figure.
5 times repeated 60 degrees rotation.

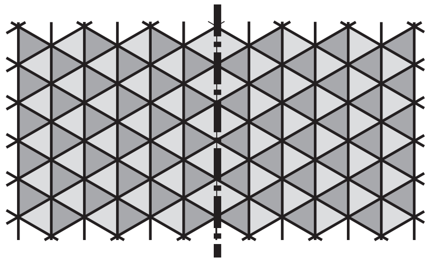


LINE SYMMETRY OR REFLEXION





Symmetry is a geometric transformation operation which is present in many natural and artificial objects. It consists of reflecting the figure about an axis of symmetry. All symmetrical points are on a perpendicular to the axis, across and at the same distance of it.

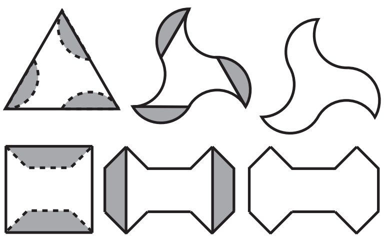


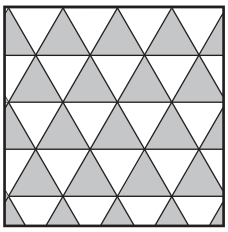
Tile transformations in tessellations: EQUIVALENCES

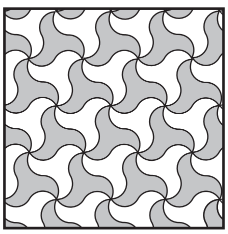
We have seen that there are three **regular tessellations** (triangles, squares and hexagons) and semiregular (there are eight), in which more than one regular polygon appear. We can also find many tessellations whose tiles are irregular polygons. And being repeated they can fill the plane (irregular triangles, rhombuses or rectangles for example).

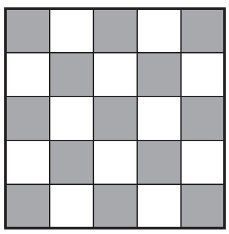
There is the possibility of altering the shape of the tile (mainly used in tessellations that only one tile, figure or module) so that the altered shape fills in the same way the plane. These employ **equivalent figures** .

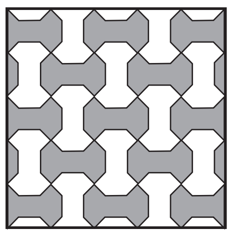
The **equivalence** is a **ratio between figures** (any plane figure) **where the original shape and the transformed have the same surface area**.











As we can see in the pictures above we have obtained an **equivalent figure** of the triangle (called **Nazari birdie**) and another figure (**Nazari bone**) equivalent to the square. We got the new figures cutting and pasting the cuts in a different places.

These cuttings or transformations atend to the laws of isometries (translation, rotation and symmetries). There are various procedures methods to obtain an equivalent figure, applying isometries, also tessellating the plane as the original figures. The Arabs and M.C. Escher were experts on this topic.

Proportion: The relationship regarding dimensions between two parts or between a part and the whole.
Ratio: Same as proportion. It is the relationship between two quantities, normally expressed as the quotient of one divided by the other.
Golden ratio: It is a proportion between two lengths, so dividing a line in two parts, the longer part divided by the smaller part is also equal to the whole length divided by the longer part. These two dimensions form a golden triangle and the result of the division is always 1,618. This ratio is present in art and nature constantly.

Equality: It is he relationship between two figures with the same shape and dimensions. A polygon is Equal to another if their sides have the same lengths and the vertices the same angles.
Similarity: It is the relationship between two figures with the same shape but different dimensions. Similar polygons have the same angles in their vertices but the lengths of their sides are different but proportional.
Equivalence: Two geometric figures with the same area, having or not the shame, are equivalent.

Rotation: A movement around a center which describes a circular path. A rotation is determined by a rotation angle in degrees, and can be in clockwise (positive) or counterclockwise (negative) direction, in relation with the center.
Symmetry: A relationship between two figures or bewteen the points of a figure.
Line symmetry: Also eflexion. Each point has its symmetric on a perpendicular to the axis, at the same distance but on the other side.
Point symmetry: Each point has its simmetric on a line through the center, at the same distance but on the other side.
Translation: It is a motion in which every point follows a straight line in a particular length. A translation is determined by a vector whith a length, slope and a direction.

Copying polygons methods: There are different methods in order to draw an equal polygon to another.
Angles and sides copying: Copying the angles and transfering the dimensions to each side of the copied angles to keep copying angles and sides.
Using coodinate axes:Setting two perpendicular axes aside and below the polygon they help us locating every vertex projecting every point perpendicularly to the axes, so making a new couple of perpendicular axes we can copy every distance and projection ray in order to form a new equal polygon.
By triangulation:Any polygon can be decomposed in several triangles. So copying every triangle adjacent to each other is easy taking with the compass the lengths of every side in order to re-compose the original polygon.

Scale: It is the ratio between the real size of the object and its depiction on the paper. It is always expressed with two numbers; Drawing dimensions / Real dimensions
Full scale: 1/1, the object and the drawing have both the same dimensions.
Extension scale:The object is depicted bigger than in reality. So 2/1 means the object is twice bigger in the paper than in reality, 3/1 means the object is three times bigger on the paper than in reality, etc.
Reduction scale: The object is depicted smaller than in reality. So 1/2 means the object's dimensions are half of the actual sizes on the paper, 1/3 means the object is depicted three times smaller on the paper than in reality.
Graphic scale: It is the representation of the scale on the paper. A graphic scale can be used as a measure to set the transformed legths on the paper.

Alberti's veil: It is a technique to copy reality of other two dimensional representation onto a two dimensions medium such as a paper, a canvas or a wall. It consists on fragmenting the full object of the representation into squares, creating a grid and copying the content of every square forming the grid in a similar grid set over the medium.
Grid: A net of straight lines that intersect each other to form a series of squares or rectangles.

Tile:In visual arts they are design units that are repeated to form a structure. Tiles can hold any kind of shape and contents and they can generate different types of grids. The repetition of the tiles is leaded by the plane transformations such as reflexion, the two types of simmetry and rotations.
Grid, Tiling or Tessellation: A drawing or depiction formed by one or several units which are repeated following a pattern or not in order to create a unique design. It is very common in everyday life being present in tiles, architecture and clothing design.
Simple grid: It is obtained from the repetition of one or several modules.
Composite grid: It is obtained from overlapping two or more single grids.
Tessellation: A particular case of modular structure consisting in filling in the plane or a flat surface with pieces, performing as modules, without leaving a blank spot.