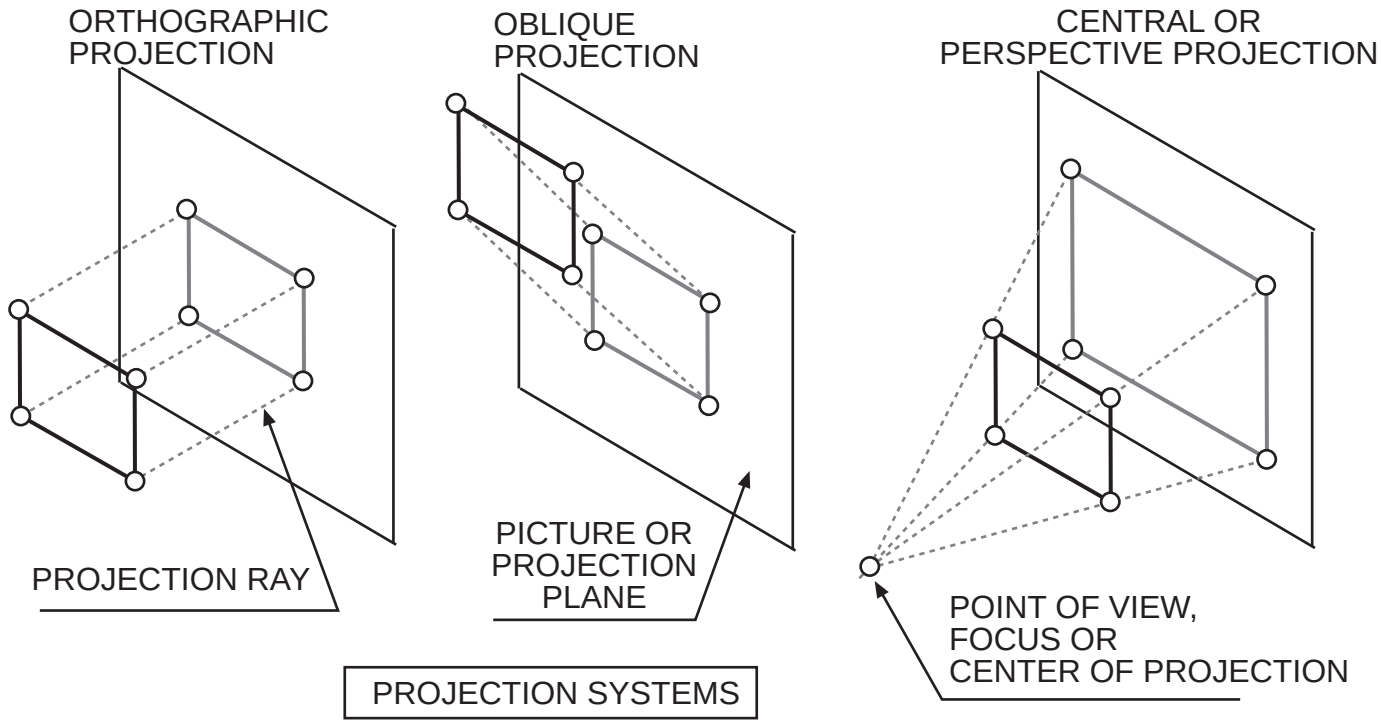
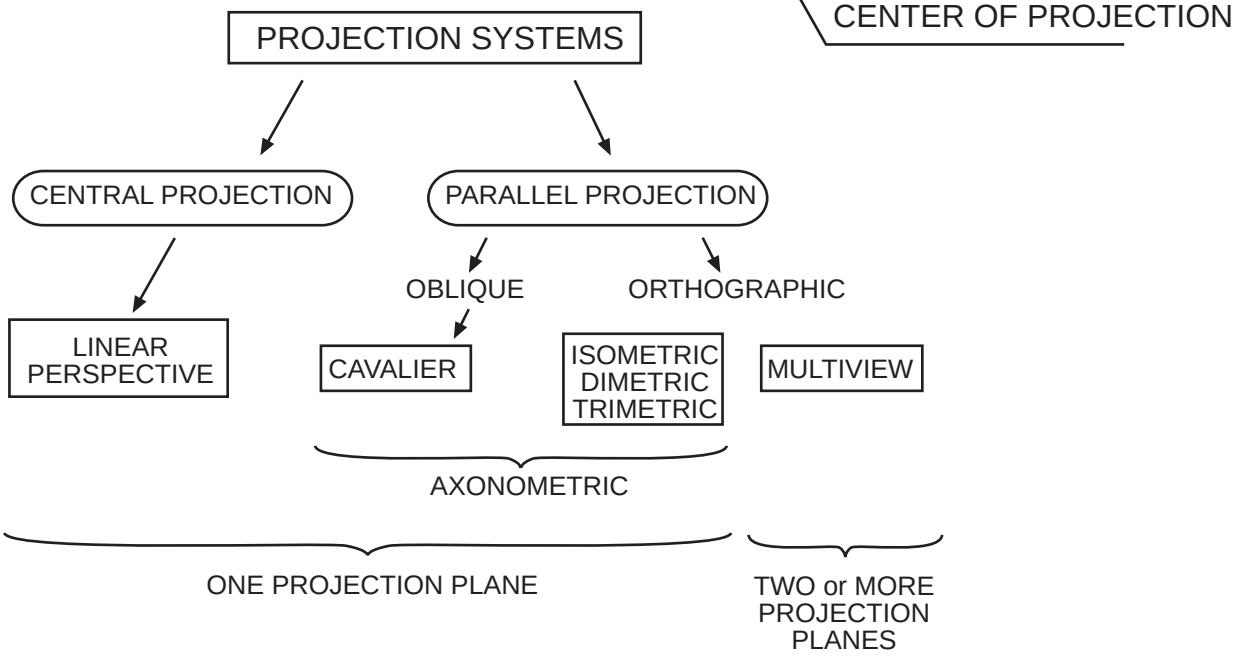
Projective Geometry

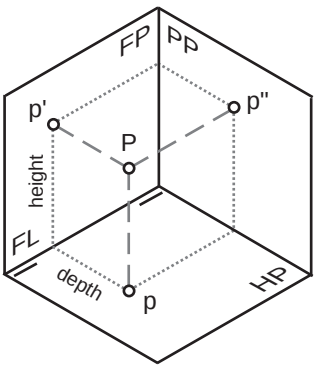
When drawing flat figures we use the **Euclidean geometry**. But when we need to **draw volumes or spaces**, we always draw on **flat** surfaces, we need to use **descriptive geometry**, also called **projective geometry**. This type of geometry was firstly used by architects and artists during the Reinascence, and later developed by mathematicians such as the French **Garspard Monge** or **Rene Desargues**.
This type of geometry is based in the concept of **Projection**. A Projection of a point is the resulting intersection point of a **projective ray**, or projection line, through the point with a **projection plane**, sometimes also called **picture plane**.

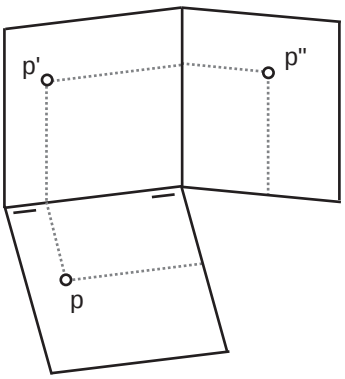
There are two types of **projections**: The central or **perspective** projection, whose projecting rays come all from a point or focus and the **parallel** projection, whose **rays** are all parallel. Rays are always projecting elements such as points, lines or objects on the projection **plane**.
Regarding the parallel projection, it can be classified in **Orthographic** or **Oblique** depending on which angle size the rays intersect the picture plane.
Parallel, orthographic or oblique, are used for the **axonometric** systems , which has one single picture plane, and also for multi-view system, which has two or more projection planes. Central projections are used to draw **linear** perspectives.

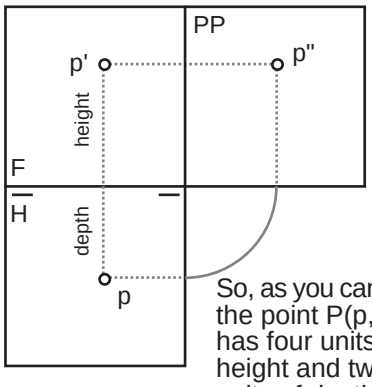




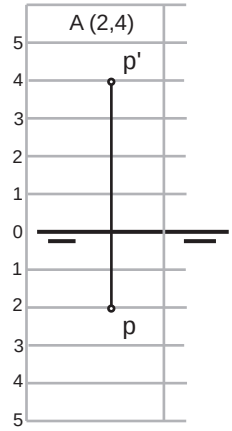
In the drawings below we see first the **multiview system projection planes** forming 90º with each other. In the next drawing we see the three planes **unfolding**. And in the third illustration we can notice how the planes end up unfolded over the same plane which is, for our practical purposes, the paper.







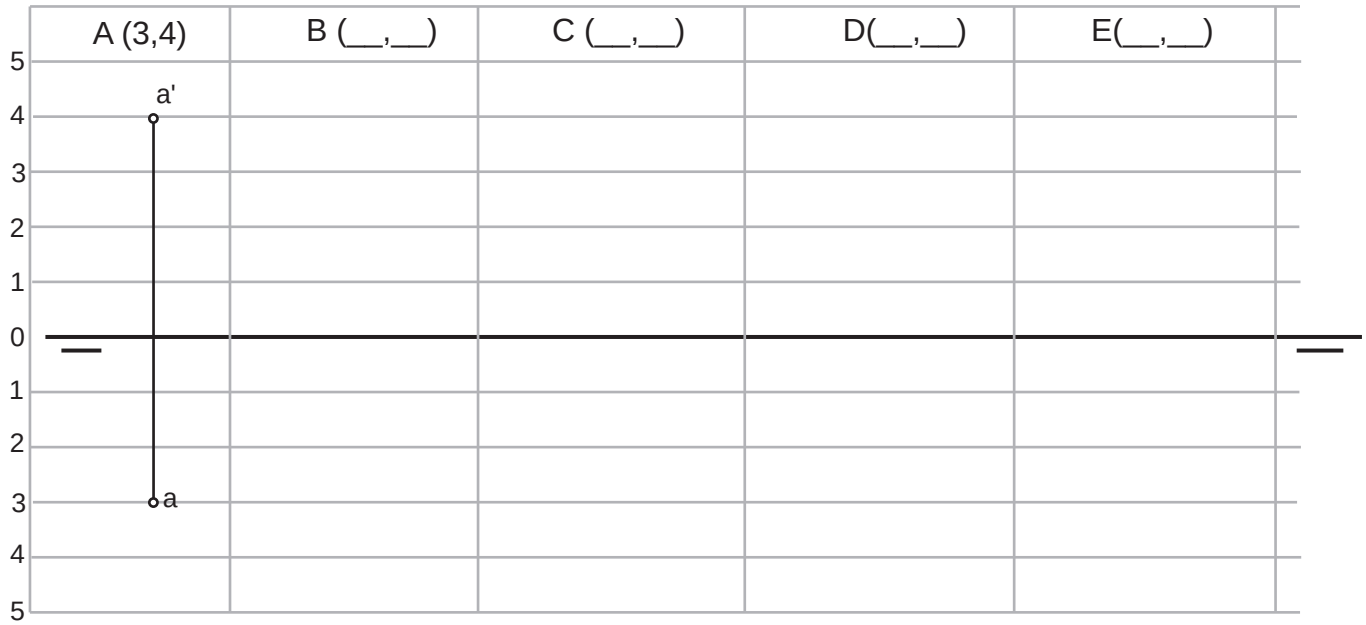
So, as you can see the point P(p,p') has four units of height and two units of depth



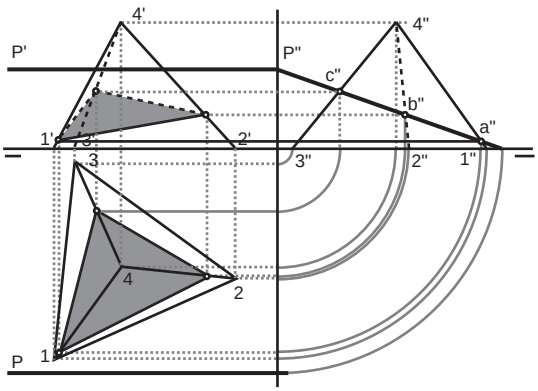
If we look at the last picture, we can see how the distance from the FL (**Fold Line**) to the **horizontal projection** of the point (p) corresponds to the **depth**, while the distance between FL and the **vertical projection** of the point (p ') represents the **height** of the point. Height and depth are depicted and correspond with other views also on the auxiliary profile plane (PP), which gives us a **third view** of the point.

ACTIVITY

Find a partner. Each of you will locate the **two projections** of a couple of points (A and B or C and D). After that tell your partner the coordinates (height and depth) for both points . Then switch the roll with your partner and at the end check both partners have the same drawings.

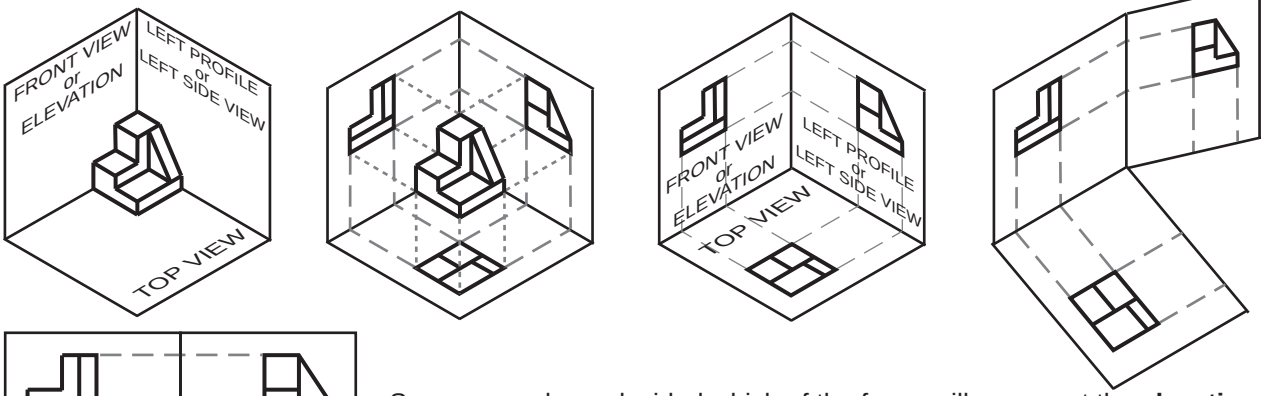


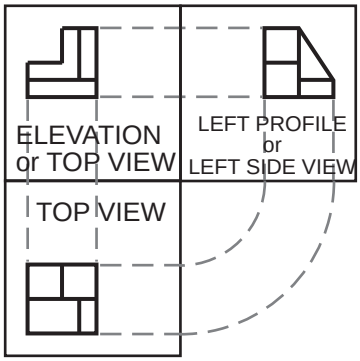
So in this page we have studied the basic concepts and the fundamental operations such as locating or depicting points in the space using the orthographic projections multiview system.
This system is used frequently by architects, engineers and designers to make plans for their creations.
As we can see depicting a point is a simple operation. This can get more complicated when we make advanced or more complicated problems.
On the right we can see the three views of a pyramid intersected by a plane parallel to the fold line.



SOLID MULTIVIEW PROJECTIONS

We have seen the operation and basic mechanics of multiview projections system. Its main use is to depict a design using the views system. Below there is a sequence in which you can see why the **position and orientation** of each **view**. In this case we have represented **top, front and left profile views** (which is always to the right of the front view).



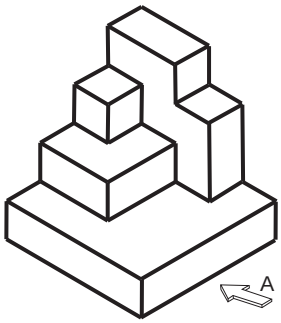


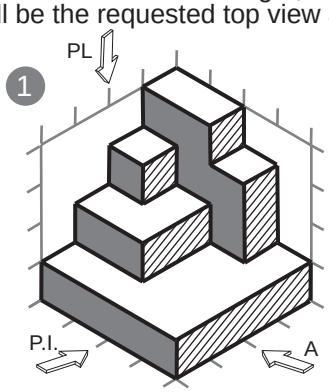
So once you have decided which of the faces will represent the **elevation (front view)**, **top views (plan)** and **profile (side view)** will be conditioned to that elevation. This is decided or requested (in tasks formulations) in the first illustration of the sequence with an arrow.

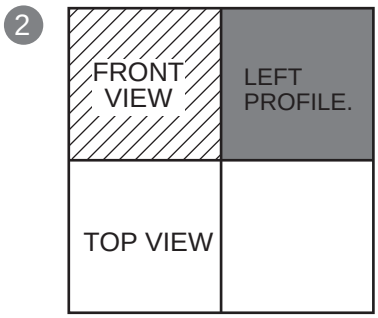
But in practice, this process will take place mentally and we actually have to decide by ourselves the front view or elevation, or attend the exercise formulation and make the process of the first three illustrations mentally.

Nextwe will show a formulation to solve it in a more practical and similar way we solve our exercises.

1 - The first thing to do is to measure the total height, width and depth of the solid to represent. We also need to mentally visualize which will be the requested top view and profile view.



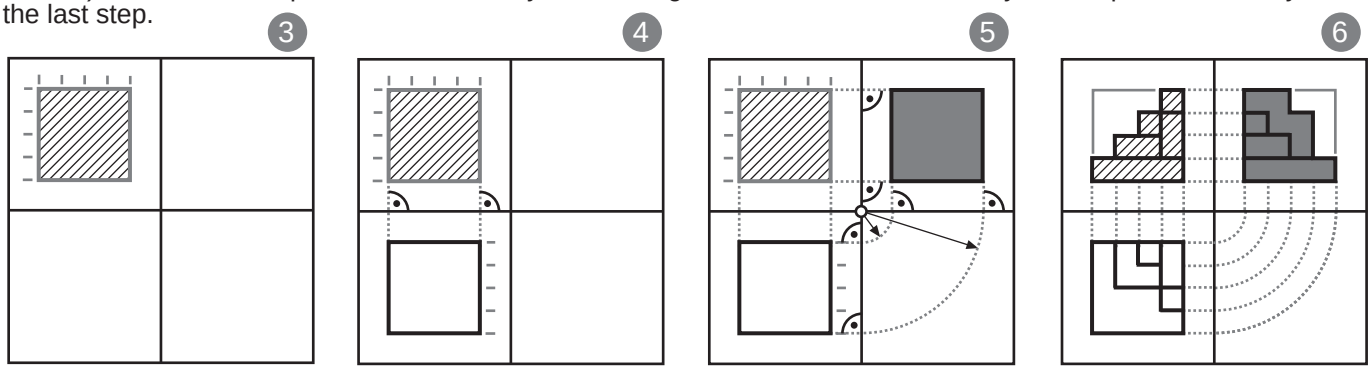




2 - Then divide the graphic space into four parts drawing a horizontal and a vertical line through the geometric center of the graphic space.

We must realize in which quadrant each view will be depicted.

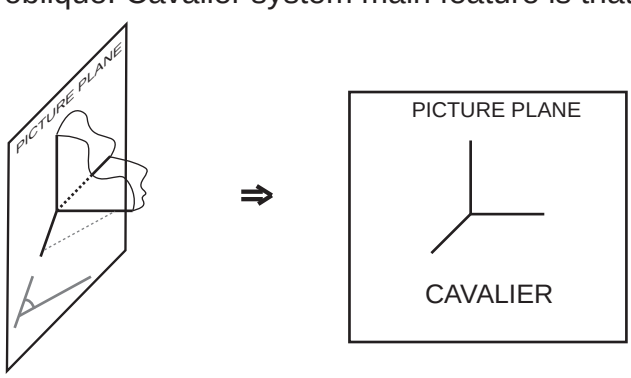
3 - In the quadrant corresponding to the front view or elevation (striped) height and draw the and full width and height of the piece according to the measures we have taken in the past.
4 - We drop the width from the front view to the part corresponding to the top view area. We will have already measured its depth and thus attending to the scale we will place it so that the top view outline gets centered .
5 - With the measures we have already set in the top view and the profile we build the left side view. This is done by drawing horizontals from the elevation and taking the depths from the top view to the vertical line and then rotate them with center at the intersection of the vertical with the horizontal which divide the spaces for views. Reference lines (widths, heights and depths must always be parallel to each other and perpendicular to the two lines (horizontal and vertical) that divide the space for views. Many exercises give us these "boxes" already built to proceed directly with the last step.

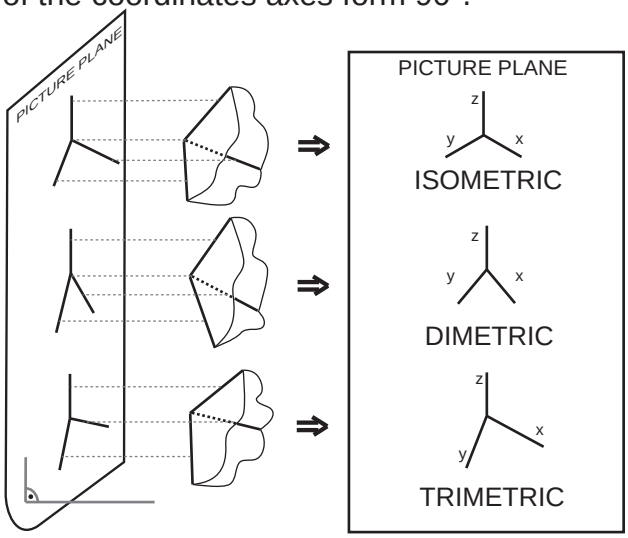


6 - We draw inside the front view or elevation and then, following the same widths draw the top view where we must place all depths from the various parts of the piece. Moving heights from the front view or elevation and depths from the top view we can, without measuring anymore, draw full profile.

The word **Axonometric** comes from the Greek terms Axo (axes) and Metrics (measures). Axonometric are those depictions of objects or pictures that have been carried out through three axes depicting the three dimensions.

To obtain the **axonometric axes**, the three intersecting **coordinates planes,** which intersections are the coordinates axes, are projected on the **picture plane**. **Projections** are always **orthographic** and **parallel**. Sometimes projections are **oblique**, these form an **angle** different than 90 ° with the picture plane, in that case the **axonometric system** is called **Cavalier** and it is not orthographic but oblique. Cavalier system main feature is that two of the coordinates axes form 90º.





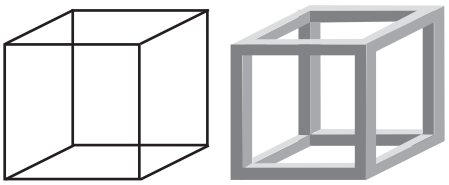
Getting focussed on the orthographic axonometric systems, depending on the angles the axes form with each other they are called in different ways.

We call the drawing **ISOMETRIC** if the axes are projected onto the picture plane (the paper) forming three equal angles (120 º). If two angles are equal but one out of the three is different we say the drawing is in **DIMETRIC** system. And if each angle has a different size it will be called a **TRIMETRIC** system drawing.

IMPOSSIBLE OBJETCS AND SPACES AND AXONOMETRIC SYSTEMS

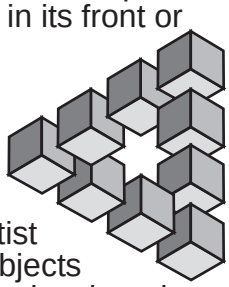
Pictorial systems make possible to **draw the space and volumes** which have **three dimensions** onto a paper or any surface that has only **two dimensions**. Descriptive or projective geometry is the branch of science that studies and set the rules for these pictorial systems.

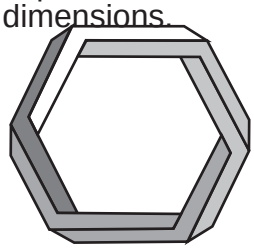
Since the 19th century there has been authors who researched the pictorials of impossible objects or spaces.



One of the earliest drawings we find is the **Necker's cube** in 1834. It is a **cavalier system** representation of a Cube which **faces or edges** are impossible to be thought in its front or in its back.

Reutersvard was a Swedish designer who drew a triangle in 1934, made with isometric system little cubes, that seems to be impossible to **recreate** in the true three dimensions.



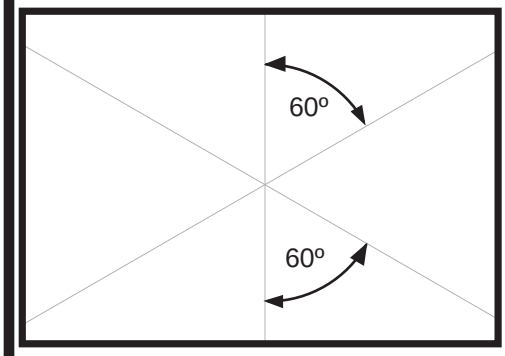


Roger Penrose is a scientist who likes the impossible objects and recreational maths. He developed the Reutersvard triangle idea writing along with his father in 1958 about a kind of **impossible three-dimensional polygons** which sides seem to be **twisted** along their perimeter.

Belvedere tower. MC Escher. 1958

In the same year **Escher** made his artwork called **Belvedere Tower** which looks like a twisting two floor building and shows a character holding a neckers cube.

ISOMETRIC DRAWING

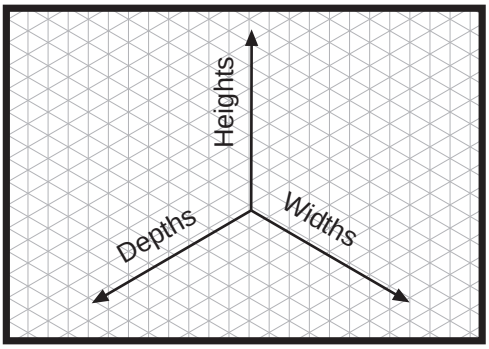


PREPARING THE ISOMETRIC GRID:

1-Draw a vertical line, choose a middle point and draw through it the two lines forming 60 ° with the first one.

2 - Mark the vertical line every 0.5 cm. and through each mark draw parallel lines to both 60 ° oblique lines.

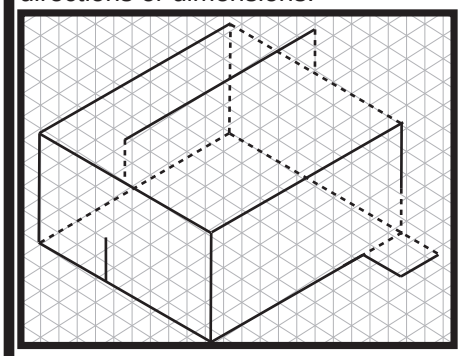
3- On the oblique lines intersections we draw vertical lines to complete the entire graphic space.



IF DESIRED THIS OPERATION CAN BE CARRIED OUT WITH THE PAPER SHEET ARRANGED VERTICALLY.

DRAWING ON THE GRID

The lines perform as the three main directions: Height, Width and Depth. All the drawing must be based on these three directions or dimensions.

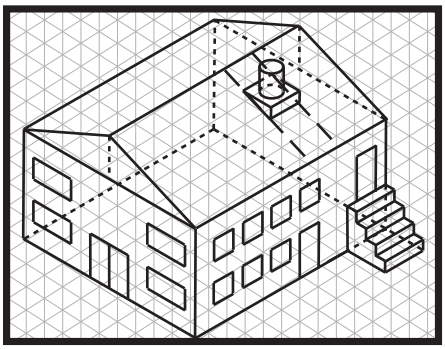


The drawing is always started drawing the full top view following the depths and widths directions.

Even though many times part of it won't be shown in the final drawing.

From the top view we start raising heights.

In Isometric drawing we must draw a lot of auxiliary lines that won't be shown at the end but are needed to make a right drawing.



If you plan to do some steep plane such as a roof, it must be marked the line that sets the peak of the roof and the lower limits of it, so from there you just draw the slope defining the plane, these last lines, as they are steep do not follow the isometric directions.

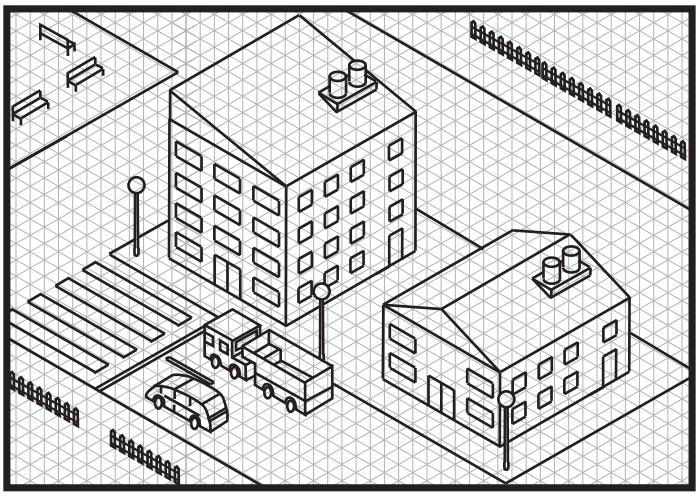
FROM THE MULTIVIEW PROJECTIONS TO THE ISOMETRIC DRAWING

There are two main ways to draw a solid from its multiview projections.

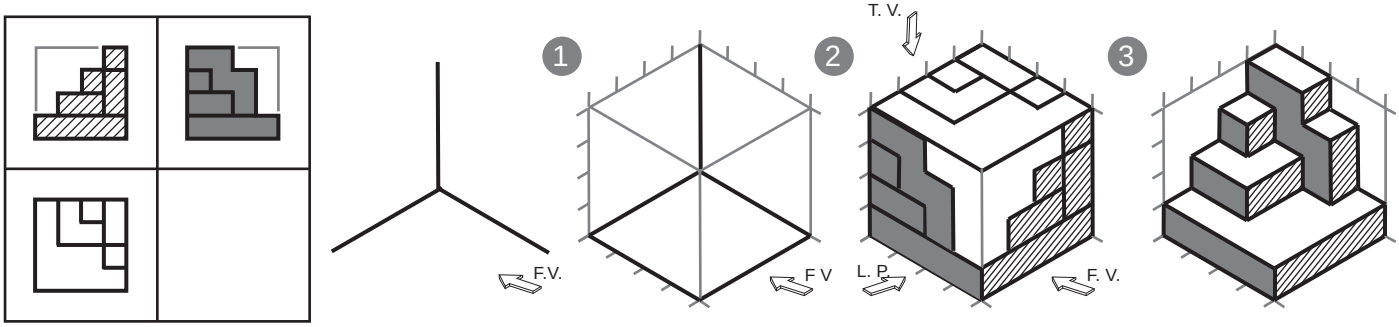
The most common method is about drawing first the top view and raising from it its heights as they are shown in the front and profile views.

But we can also draw a "transparent isometric box" with six faces adjusted to the solid showing its main dimensions (height, width and depth).

Once this is done we draw on each of the box faces its corresponding view to relate the elements from the different views in order to depict them tridimensionally.



This second method is perharps more efficient or practical if the solid is not simple. An advantage of this method is that the "box" side faces areas that do not show part of the solid can help us to remove part of the box and then we will know that there is nothing to draw there. This mental process when working with 3D software is called "to extrude" or "extrusion".

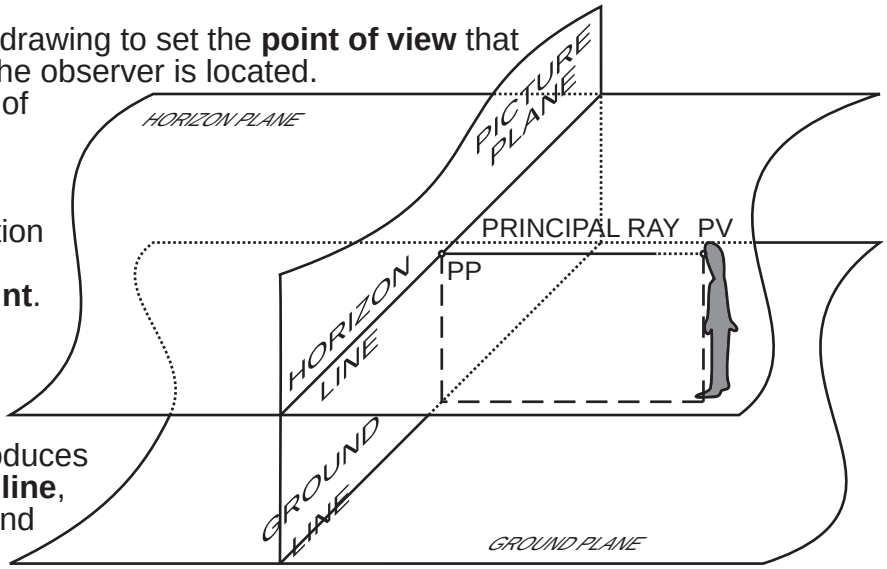


In order to draw a perspective we need to establish the elements that will make it possible, they are the following:

The **ground plane** is a flat surface where everything stands, it is horizontal and represents the floor. Perpendicular to this plane we need a **picture plane** where the projection or drawing will appear through the intersection of the **visual rays** with this plane. The intersection line between the picture plane and the ground plane is the **ground line**.

It is also necessary for obtaining a drawing to set the **point of view** that represents the exact place where the observer is located.
A perpendicular line from the point of view to the picture plane is called **Principal Ray** and determines the distance between the picture plane and the point of view. The intersection point on the picture plane with the principal ray is called **principal point**.

The point of view also determines, at its same height, the **horizon plane** which is parallel to the ground plane. This plane produces over the picture plane the **horizon line**, always parallel to the ground line and through the principal point.

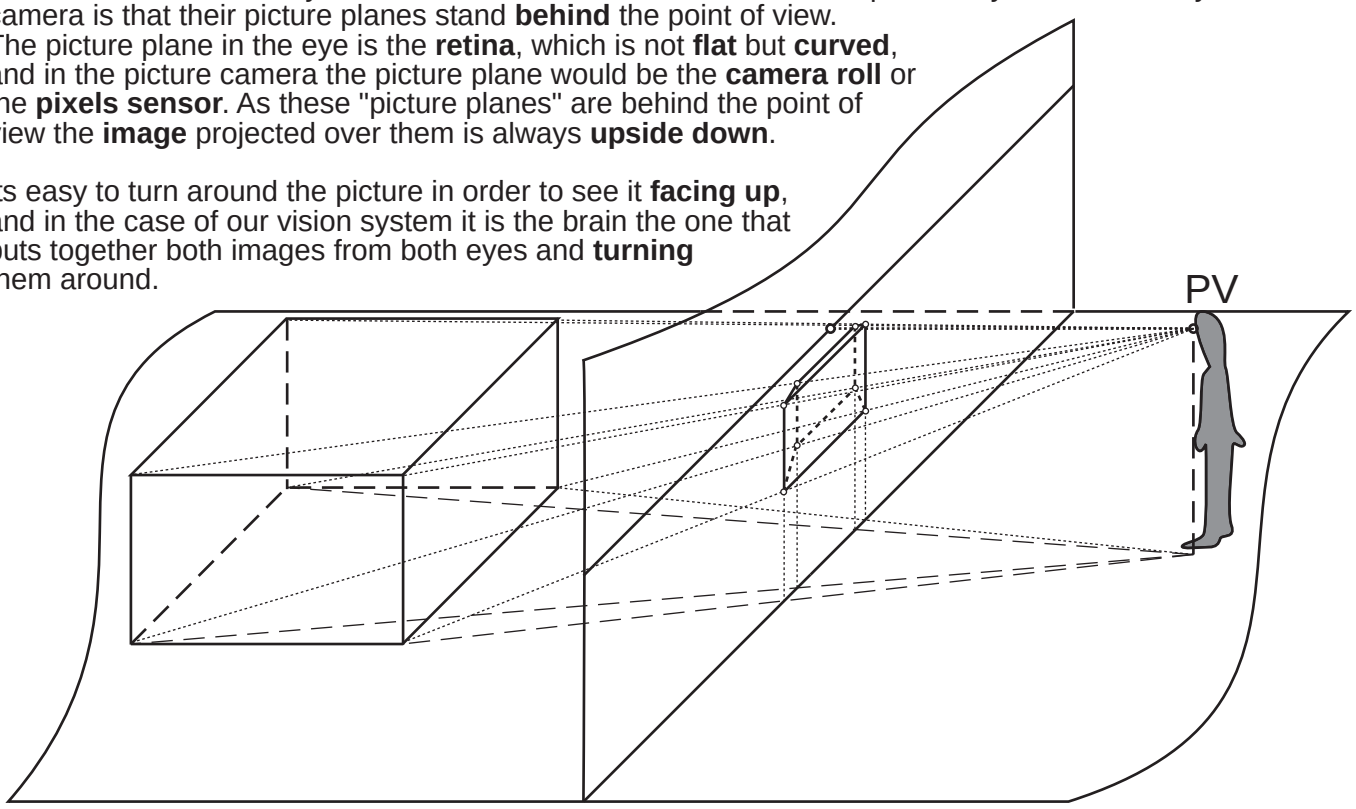


LINEAR PERSPECTIVE AND HUMAN VISION

Linear perspective, unlike other **pictorial systems** working with **parallel** projection, parallel lines, depending on their **position** in relation to the **projection** plane, are represented as **converging** on **vanishing** points.

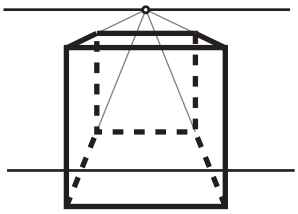
Linear **perspective** is the pictorial system whose **drawings** are more likely to the human **vision**. The projecting **rays** coming from one point are very similar to the eye's **visual** rays, the photographic **camera** or the camera **obscura** systems. One of the differences between this pictorial system and the eye or visual camera is that their picture planes stand **behind** the point of view.
The picture plane in the eye is the **retina**, which is not **flat** but **curved**, and in the picture camera the picture plane would be the **camera roll** or the **pixels sensor**. As these "picture planes" are behind the point of view the **image** projected over them is always **upside down**.

Its easy to turn around the picture in order to see it **facing up**, and in the case of our vision system it is the brain the one that puts together both images from both eyes and **turning** them around.

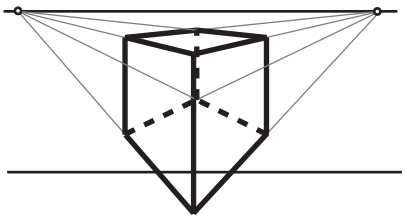


The three basic types of linear perspective are;
It is called "One point perspective" when there is one single **vanishing point** and in order to get that perspective the object, the picture plane and the **point of view** need to be set in an **ortographic (parallels and perpendicullars) layout**. "Two point perspective" is the term used to refer to perspectives with two vanishing points and in those cases the objects are set up **oblique** to the **picture plane**. And "three point perspective" takes place when the object is set up oblique to the picture plane and the point of view needs to be shown in a much higher location than the ground plane.

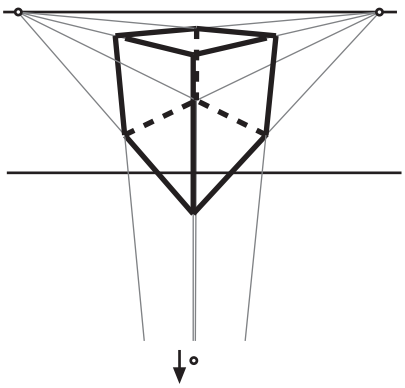
ONE POINT PERSPECTIVE:
1 vanishing point



TWO POINT PERSPECTIVE:
2 vanishing points

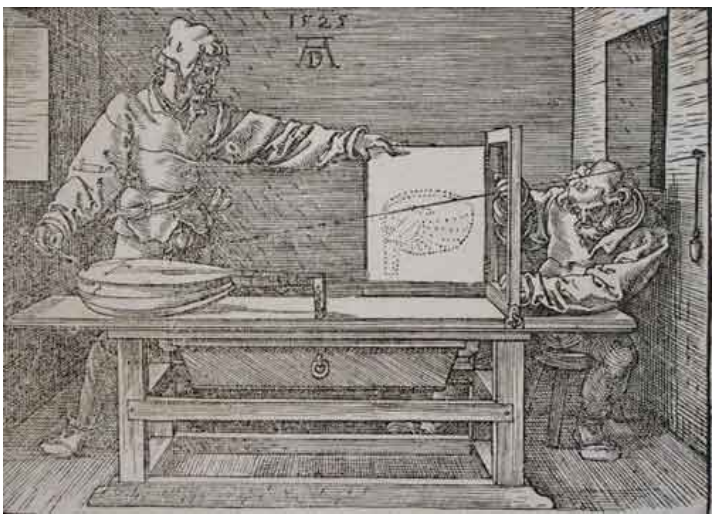


THREE POINT PERSPECTIVE:
3 vanishing points



Here you can see three pictures made by Albert Durer, who was a German humanist. These are three woodcuts done in 1525 in which Durer showed some tricks the painters used to obtain better drawings. These drawing machines are based on the Linear perspective knowledge.
Some of these machines use strings that work as visual rays and can show over a paper that performs as a picture plane the projections for the drawing. They have different appearances depending on the type of scene or the objects to be drawn. They set the point of view, and the picture plane with different arrangements that condition the roll of the artist.
In some other drawing machines the artist has to look through a peephole and so the artists can see the objects or the scene projected over the surface to draw.

Fig. 33. Albrecht Dürer, "Portrait." Woodcut. Collection Centre Canadien d'Architecture/Canadian Centre for Architecture, Montréal.



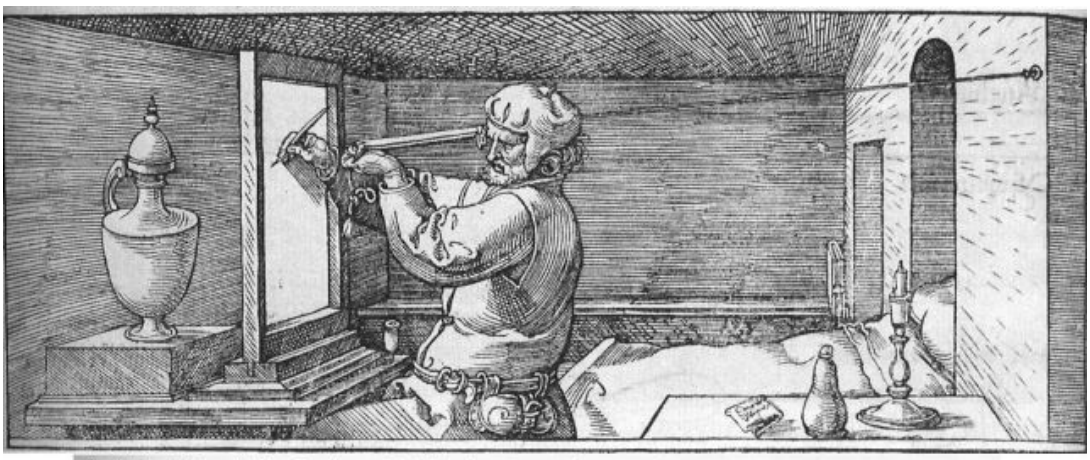


Fig. 32. Albrecht Dürer, "A Man Drawing an Urn." Woodcut. From *Underweysung der Messung*. All rights reserved, The Metropolitan Museum of Art, Harris Brisbane Dick Fund, 1941. (41.48.3).

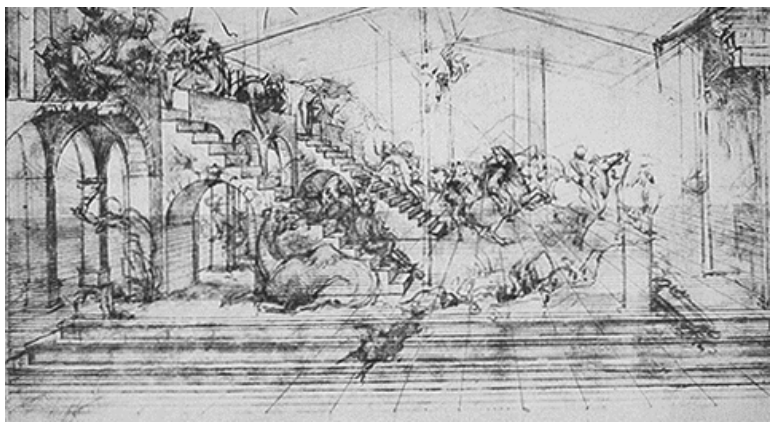
PERSPECTIVE AND ART

Linear perspective has been known and used since the Reinascence era. And on and on it has been used by artists till current days.

Antonio López is one of the most famous **hyperealist** Spanish artist. He is very interested in the **light effect** over the objects and scenes but also in perspective and **points of view**. This artist has drawn multitude of perspectives of Madrid some of his artwoks took him years to be completed as "Gran Via", painted between 1974 and 1981, showing a crossroad in Madrid really early in the morning so there are no people and no vehicles.

Richard Estes is an American hyperartist who also uses linear perspective for most of his artworks. His artwork usually shows **urban landscapes**. "Paris street scene", painted in 1975, is one of his paintings which represents the essence of his artwork very well. Estes shows in many paintings **reflected images** over mirrors or glass surfaces of the scenes.

Leonardo Da Vinci was a **humanist** whose most famous aspect was to be an artist. One of his most representative perspectives is **"The Adoration of the Magi"**. This artwork is a **sketch**, made in **1485**, which still shows the auxiliary **vanishing lines** and their **vanishing point**.



Stephen Wiltshire is an autist British artist who, despite his disabilities, has the quality of drawing **architectural elements** or urban landascapes **from his memory** with a very **realistic appearance**. He is able to visit places for first time and, with a short ride in a helicopter and a quick look to the town or place, he learns the images **by heart.** Then he can take a while putting them onto the paper.
He has ben invited to draw cities from all over the world and he only uses a **black ink pen** and sometimes some colors. He has been honored in his home country and he opened an **art Gallery** in 2006.

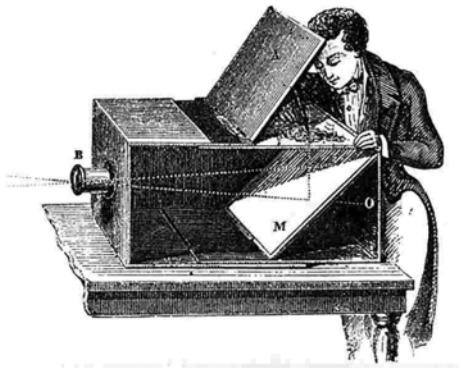
PROJECTIONS: PHOTOGRAFIC CAMERA, HUMAN VISION AND CAMERA OBSCURA

Camera obscura consists on a colored box in the inside with black, so the planes that form its **faces** don't reflect the light. In the middle of one of its faces there is a little hole that lets the **rays of light** get in the box so they get projected on the oposite wall, forming the **image** that points the hole. **Photographic** digital and roll cameras work the same way.

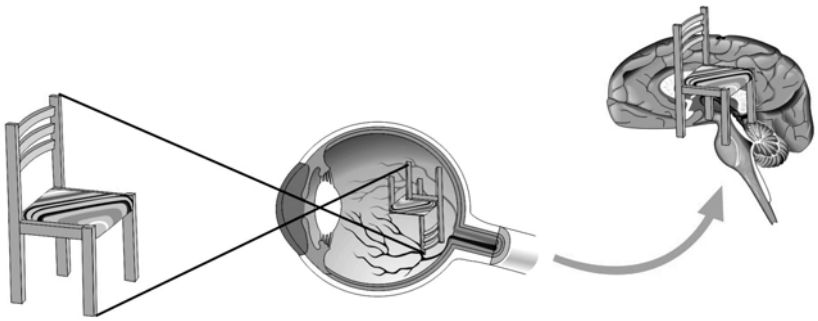
Clasic roll cameras use chemical products to **print** the light over the picture and that way the artitst were the ones who could carry out the printing task. Artists like **Caravaggio**, **Vermer** and **Canaletto** were thought to use cameras obscuras as a **tool** which helped them to draw . They would use big rooms as **huge** cameras obscuras in which they would fit the **characters** and objects to compose the scenes.

In a camera obscura the point of view is in between the scene or object and the picture plane and this arrangement leads the image to be **projected** upside down. Same thing happens to photography.

Same arrangement of the elements takes place in **human vision** in which the eye works exactly as a camera obscura. The rays of light gets into the eye through the **pupil** and gets projected **upside down** over the **retina** which performs as the picture plane. The retina is not a flat surface as every other artificial picture plane but spheric which causes some curved perceptions of some vanishing lines.



Camera Obscura: Source Wikipedia



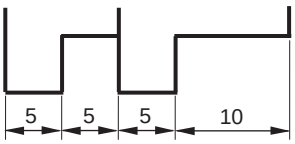
Human Vision: J.A. Bermúdez. Source "Banco de imágenes y sonidos" INTEF

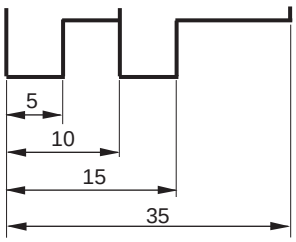
DIMENSIONING

Dimensioning consists on writing down the meassures of every part of the objects or pieces designed through any of the pictorial systems in order to be built. Manufacturers have to look at the dimensions to build the objects in a determined size. There are different elements used for dimesioning a sketch and there is also some dimensioning systems. You can take a look at the glosary before you start this activity to learn more.

The dimensions arrangement is conditioned upon the manufacturing process and the piece's role. There are two main ways to dimension a drawing:

Aligned system: Dimensions are arranged next to each other in one single level.

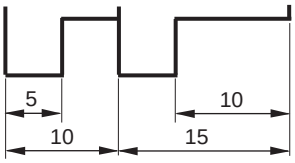




Parallel system: There are several levels with dimensions.
The longer dimesions stay underneath the shorter ones, closer to the drawing.

But knowing these two systems we can find another:

Combined system: This type is the most commonly used and it uses both, aligned and parallel systems of dimensioning.



PROJECTIVE GEOMETRY

Projection: The projection of a point is another point which is the intersection of a projecting ray, that passes through the original point, with the plane of projection.
Projection plane: a plane in which the elements are projected through the projecting rays or lines. Sometimes it is called picture plane because it is where the drawing is obtained.
Depending on the origin of the projection rays:
Parallel projection: A projection type in which the projecting rays are parallel.
Perspective projection: A type of projection in which all the rays are projected from one point or focus.
If the projection is parallel there are two types depending on the slope regarding the projection plane:
Orthographic projection: Is a type of projection whose rays are perpendicular to the projection plane.
Oblique projection: Is a type of projection whose rays are not perpendicular, they are oblique, to the projection plane.

MULTIVIEW PROJECTIONS SYSTEM

Multiview System: It is a system of representation that uses orthographic parallel projections to describe the elements represented in two projection planes perpendicular to each other, called Frontal plane (FP) and horizontal plane (HP).
Fold Line: The line of intersection between the two projection planes .
Elevation or Front view: The projection of the figure on the frontal projection plane .
Top view , Plan or horizontal projection: The projection of the figure on the horizontal projection plane.
Profile or side view:: It is an auxiliary view which is used to describe better the figures. It may represent the right side view, which is shown on the left of the front view, the left side view which is presented on the right the or both profiles if necessary.
Depth: It is the distance between the points and the vertical plane of projection.
Height: It is the distance between the points and the horizontal plane of projection. It stands for "how high".

AXONOMETRIC:

Orthographic axonometric system: It is a type of pictorial system consisting in three coordinates planes which form three axes that arrange the three dimensions on the paper.
Isometric: It is a kind of orthographic axonometry whose axes form equal angles of 120º with each other.
Dimetric: A kind of orthographic axonometry whose axes form two equal angles and a different one.
Trimetric: It is a kind of orthographic axonometry whose axes form three different sized angles.
Coordinates origin: The point where the axes of the system meet.
Cavalier system: is a type of oblique axonometric system which always shows a right angle between two of its axes. The other axis usually shows 135º with the other two.

DIMENSIONING

Dimensioning tells us the acurate measures for every part of any piece. They are used in axonometric as well as in multi view drawings.
Dimension lines: Parallel lines to the edge lines of an object. They have arrows or terminators on their endpoints.
Dimension Values: Numbers located over the dimension lines that express the true lenghts of any part in an object
Extension lines: Thin lines which are perpendicular to the object edges as well as to the dimension lines, they are used to show clearly which parts of an object a dimension is referring to.
Symbols: They are used in dimensioning for giving additional information in a short way about the dimension of an object. R is used for Radius, a circle with a crossed line for diameter or a little square before the value to tell that part has a squared shape.

LINEAR PERSPECTIVE

Linear perspective: A pictorial system based on the perspective projection of an object on a projection plane. Its main features are the **vanishing lines**, which are lines converging on the **vanishing points**.
Point of View (PV): The focus for all the visual rays (projective lines) to the object points to be projected (drawn) on the picture plane. It is the center of projection.
Picture plane: The plane where the projection or drawing is set. For practical purposes it is the paper over which spaces or objects are drawn.
Ground Plane: It is always perpendicular to the picture plane and it is the plane where objects are generally leaning. The distance between the point of view to the ground plane represents the height of the point of view.
Horizon Plane: A plane parallel to the ground plane and through the point of view which produces on the picture plane the horizon line.
Horizon line (HL): A parallel line to the ground line that is always at the same height as the point of view. It is the intersection line of the picture plane with the horizon plane. **Ground Line** (GL): It is the line of intersection of the picture plane with the ground plane, always parallel to the horizon line.
Vanishing Lines: These are the projection (drawing) on the picture plane of the lines which are perpendicular or oblique (not parallel) to the picture plane. Every set of vanishing lines, parallel to each other, interesect on the same vanishing point.
Vanishing points: The points, usually on the horizon line, where vanishing lines meet.
Principal point: is the point of view orthographic projection onto the picture plane, it is always on the horizon line, and the vanishing point in one point perspectives.
Foreshortening: Sometimes refered to human body and sometimes to perspective. When an object or segment is depicted smaller than its real size because of its position in relation with the picture plane. Foreshortened objects usually lay partially or totally perpendicular to the picture plane.